Scand. J. of Economics 106(3), 511–532, 2004
DOI: 10.1111/j.1467-9442.2004.00380.x

Decreasing Impatience: A Criterion for Non-stationary Time Preference and "Hyperbolic" Discounting

*Drazen Prelec**

MIT, Cambridge, MA 02139-4307, USA
dprelec@mit.edu

Abstract

Despite recent interest in hyperbolic discounting, there has been little discussion of exactly what property of time preferences is instantiated by hyperbolic or quasi-hyperbolic functional forms. The paper revives an earlier proposal in Prelec (1989) that the key property is Pratt–Arrow convexity of the log of the discount function, which corresponds to *decreasing impatience* (DI) at the level of preferences. DI provides a natural criterion for assessing the severity of departure from stationarity in that greater DI is equivalent to more choices of dominated options in two-stage decision problems, as well as greater convexity of the log of the discount function. Inefficient choices may arise as intentional precommitments, or as unintended reversals of preference by "naïve" agents.

Keywords: Time preference; hyperbolic discounting

JEL classification: *D*91

I. Introduction

Few economic hypotheses have advanced so rapidly from the fringe to the mainstream as hyperbolic discounting, which now functions almost as a default option for analyzing the misbehavior of economic agents; examples include undersaving as in Laibson (1997), procrastination as in O'Donoghue and Rabin (1999a), addiction as in Gruber and Koszegi (2001), and self-confidence as in Benabou and Tirole (2002). The well-known distinguishing feature of hyperbolic agents is that they exhibit steeper discounting for time intervals closer to the present. In Laibson's (1994) influential quasi-hyperbolic simplification, the steeper discount rate applies only to the initial interval. As a result, intertemporal trade-offs shift with the mere passage of time, and plans or projects that seemed optimal yesterday need no longer

*The results presented here are partly based on Prelec (1989), an unpublished working paper written while I was a fellow at Russell Sage. I would like to thank the Russell Sage Foundation for support, and two anonymous referees for generous and constructive advice on the present paper.

be optimal today. From the theorist's standpoint, the beauty of the hyperbolic assumption is that it appears to explain flagrant violations of normative principles, such as selection of dominated outcomes or rejection of information by appealing to an aspect of preference, while leaving the agent's underlying rationality intact; see Carrillo and Mariotti (2000).

Given the interest in hyperbolic modeling, it is perhaps surprising that there is no rigorous definition of what property of time preference is instantiated by hyperbolic or quasi-hyperbolic functional forms. These forms are acknowledged to be more or less convenient simplifications, but what exactly are they a simplification of? Here, I revive an earlier proposal in Prelec (1989) that the critical property is Pratt–Arrow convexity of the log of the discount function (also called decreasing impatience). The virtue of this criterion is that it sorts functions according to their tendency to promote selection of dominated (i.e., inferior and more delayed) outcomes in two-stage dynamic decision problems. The dominated outcomes—which cannot be optimal from the perspective of any "temporal self"—are chosen for self-control reasons. At an earlier point in time, the agent realizes that the superior, dominat*ing* outcome cannot be obtained, in the absence of pre-commitment devices, and hence settles for the dominated outcome as a second best.

This paper is written from a psychological perspective, with the aim of understanding the psychophysics of time discounting independent from specific economic applications. Section II introduces the core supporting intuitions for hyperbolic discounting (and is taken verbatim from Prelec (1989)). Readers familiar with hyperbolic discounting may freely skip to Sections III and IV, which prove an equivalence between decreasing impatience and the inability to obtain an efficient (i.e., undominated) outcome in simple dynamic settings.

II. Intuitive Arguments for Hyperbolic Discounting

Consider a *temporal prospect*, which is a finite sequence of dated outcomes $(\boldsymbol{x}, \boldsymbol{t}) = (x_1, t_1; \ldots; x_n, t_n)$, such that outcome x_i will occur with certainty after delay t_i if the prospect is accepted. The stationarity axiom states that preference between two prospects does not change if all dates are translated by a constant amount, which is to say if the two prospects are contemplated at a closer or more distant vantage point. From the work of Koopmans (1960), we know that this condition is needed to insure constant (or "exponential") discounting, and from the earlier work of Strotz (1955), we know that a person who does not discount at a constant rate (and who resets the zero on the discount function when the next decision arrives) will be prone to *dynamic inconsistency*, that is, he may reverse the decisions formulated in the earlier stage.

Unfortunately, both intuition and experimental evidence suggest that our preferences do not conform to the exponential discount rule.[1] It is not implausible, for example, that I should prefer a free movie ticket tonight to a free ticket plus popcorn next week, but, at the same time, prefer a ticket plus popcorn in eleven weeks to a mere ticket in ten—on the ground that the difference between ten and eleven weeks is insignificant. This type of reasoning would, of course, produce a direct violation of the stationary axiom.

A related phenomenon, first predicted by Ainslie (1975), is that people seem to be less impatient when stakes are enlarged, so that an entire sequence of intertemporal choices hinges on a single decision. If the decision in the earlier example was between two booklets of tickets (all with or without popcorn), say, then I may feel that the popcorn is suddenly worth waiting for. This, too, is inconsistent with stationarity, assuming that the ten-week delay merely shifts the date of each movie visit by a constant (i.e., ten-week) interval.

A less obvious, but perhaps more universal form of violation occurs when various unpleasant but necessary actions are postponed repeatedly. I may, for example, prefer to fix the air-conditioner, TV set or leaky faucet next week instead of now, but faced with a stark once-and-for-all decision, I would choose to fix it now rather than have it, say, permanently broken. This phenomenon has nothing to do with a high rate of time preference, *per se*. The problem, instead, is with stationarity: a preference for postponing the chore one week implies (stationarity) that I should also prefer to postpone it from week N to week $(N+1)$, which then implies (by transitivity) that leaving it unfixed is the best and fixing it now the worst of all possible alternatives.

The banality of these examples only proves that we are dealing with robust and pervasive aspects of "natural" (i.e., uninstructed) time preference. It is noteworthy, however, that all three forms of stationarity can be derived from a single assumption, namely that the discount factor for a fixed time interval decreases as the interval becomes more remote.[2] To illustrate

[1] Thus Fishburn and Rubinstein (1982, p. 681) conclude that: "[they] know of no persuasive argument for stationarity as a viable psychological assumption". Starting with Chung and Herrnstein (1967), experimental psychologists have repeatedly found non-exponential discounting in animal intertemporal choice. The ramifications of non-exponential time preferences for such diverse phenomena as impulsiveness, self-control, emotion and psychotherapeutic technique have been worked out in a series of provocative books and articles by Ainslie (1975, 1982, 1984, 1989, 1992). Evidence on human subjects can be found in Benzion, Rapoport and Yagil (1989), Chapman and Elstein (1995), Ebert and Prelec (2003), Kirby and Herrnstein (1995), Kirby and Marakovic (1995) and Thaler (1981), and is reviewed in Frederick, Loewenstein and O'Donoghue (2002). Recent applications to economic theory include Benabou and Tirole (2002), Brocas and Carrillo (2000, 2001), Gruber and Koszegi (2001), Harris and Laibson (2001) and O'Donoghue and Rabin (2001).

[2] However, procrastination does require the additional O'Donoghue–Rabin (1999a) assumption that agents naïvely fail to anticipate future change in time preference; see also O'Donoghue and Rabin (1999b, 2000).

this, Figure 1 plots the present value of four elementary cash-flow patterns, calculated according to a single non-exponential discount function, and normalized so that each cash flow sequence has value one at time zero. The third line from the top is the present value of a single positive cash flow; hence the present value function (after normalization) is the discount function itself, representing the underlying rate of time preference. The line just above it is the normalized present value of a sequence of ten identical, equally spaced cash flows, plotted as functions of the delay to the starting time of the initial flow. We note that the present value of the entire package decreases at a lower rate than the present value of the single cash flow. The more general implication is that any items whose benefits extend over time (such as consumer durables, or large cash flows) will be underdiscounted in comparison to the underlying rate of time preference.

Once we combine positive with negative cash flows, the derived discount functions become complex, and rather suggestive psychologically. The lowest

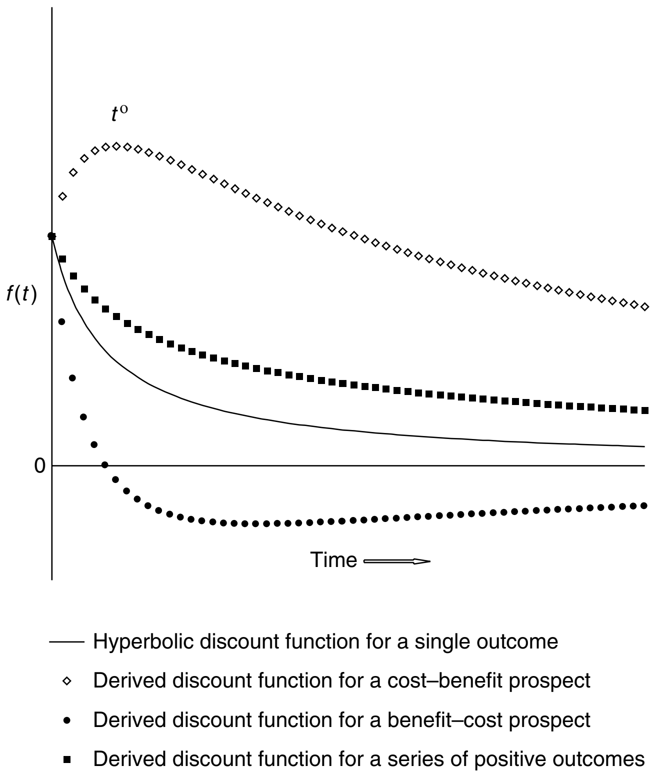

Fig. 1. The present value of four different temporal prospects, plotted as a function of the delay to the initial outcome, and normalized to have value +1 at time zero (explanation in text). The underlying discount function is hyperbolic, $f(t) = (1 + \alpha t)^{-\beta/\alpha}$

curve is an example of *hyper*-discounting, produced by plotting the present value of a cash-flow pair—an initial inflow followed by a delayed (and larger) outflow. The biphasic prospect has positive (unit) value at time zero, when the benefit is immediate, after which the present value drops precipitously—at a rate much steeper than the rate of time preference—and eventually becomes negative. Thus the prospect would be rejected if the decision was made at a sufficiently great distance. The complementary phenomenon is shown in the top curve, which plots the present value of an investment-type prospect (cash outflow followed by cash inflow). The discount rate is *negative* at first, indicating that a person would prefer to postpone the initial cost, to the optimal starting time t^{o}. There is no problem with this if one can make a binding commitment to that starting time; but if commitment is not possible, then at t^{o} there will again be reason to delay. Hence the prospect may never get accepted, even though its present value is always positive.

III. Decreasingly Impatient Time Preferences

By dropping stationarity, we seem to have opened up a Pandora's box of mutually inconsistent attitudes to future prospects, all computed on the basis of a single underlying discount function. In Figure 1, this function is the actually-hyperbolic[3] $f(t) = (1 + \alpha t)^{-\beta/\alpha}$; see Loewenstein and Prelec (1992) and Prelec (1989). Setting aside this particular form, is there a way of characterizing unambiguously whether one set of preferences, or, for that matter, one discount function, generates more dynamically inconsistent behavior, according to a reasonable definition of dynamic inconsistency? This was the motivating question for my 1989 working paper. The hope was that a criterion could be identified which would play a role analogous to the Pratt–Arrow measure for the utility function. Such criteria serve not only to sort functions, but also to clarify the key property behind the behavioral phenomenon, in one case risk aversion, in the other dynamic inconsistency. *A priori*, one might expect that the proper criterion would be independent from outcome utilities, and that it would also cleanly separate violations of stationarity from the rate of time preference.

It is important to realize that even at the level of discount functions, the simple directional requirement—viz., that there is less discounting over a time interval as the interval moves further away—is compatible with alternative criteria. One

[3] This should be distinguished from the quasi-hyperbolic Laibson–Phelps–Pollak form, as in Laibson (1994) and Phelps and Pollak (1968), $f(t) = \beta\delta^{t}$, $t > 0$, $f(0) = 1$, which has proven extremely productive for multi-period modeling; cf. Harris and Laibson (2001, 2003), Laibson (1997) and O'Donoghue and Rabin (1999a, 1999b, 2000, 2001). The actually-hyperbolic generalizes simpler hyperbolas, $1/t$ as in Ainslie (1975) and $1/(1 + \alpha t)$ as in Herrnstein (1981). It approaches the exponential function as $\alpha \to 0$. The quasi-hyperbolic is usually expressed in discrete form, as a sequence of discount factors, $\{1, \beta\delta, \beta\delta^2, \beta\delta^3, \ldots\}$.

could, for instance, compare functions by convexity, because convexity promotes higher discounting of the immediate future and lower discounting of the distant future. A second possibility is to compute the instantaneous discount rate, $-f'(t)/f(t)$, and see whether the discount rate for one function declines faster than the discount rate for another. In that case, two functions, f^* and f, would have the same degree of non-stationarity if $f^*(t) = e^{ct}f(t)$.[4] A third possibility is to compare functions by convexity of $\ln f(t)$; this is equivalent to ranking by proportional, rather than absolute changes in the discount rate. As we shall see, the equivalence classes are then given by the power transformation, $f^*(t) = f(t)^c$. All three possibilities are illustrated in Figure 2, which displays three discount functions, each of which is "more deviant" than the other two according to one particular criterion: A is the most convex, by the Pratt–Arrow measure of comparative convexity; B has the fastest drop in the discount rate; C has the fastest proportional decline in the discount rate. Of the three, which function will lead to the most dynamically inconsistent choices?

There may not be a single completely satisfying answer to this question. However, if we focus on an elementary form of dynamic inconsistency then a natural criterion does emerge. The main idea is again shown in Figure 2. The three vertical bars represent three desirable outcomes, $x < z < y$, available at fixed dates (indicated by position on the time line). At time zero, a person can go for z, or postpone the decision until time τ, when she is free to choose among all three outcomes. z is a dominated prospect, because it delivers an outcome inferior to y and at a later point in time. However, given the right "unlucky" combination of utilities, a person with discount function C may indeed choose z at $t = 0$, anticipating that at $t = \tau$ she would choose the immediate outcome x, which is even less attractive from vantage point of $t = 0$. In contrast, there are no utilities that reconcile—at those specific dates—a choice of dominated z with discount functions A and B. Indeed, over the time interval in Figure 2, this comparative relation holds generally: whenever one can reconcile a dominated choice of z with function A or B, then one can also do the same with function C. In that sense, function C promotes more inefficiency in dynamic settings.

To formalize this idea at the level of preferences, I assume that preferences $\geq$ over prospects are represented by a continuous, strictly increasing utility function ($u(0) = 0$) and a strictly decreasing discount function $f(t_i)$($f(0) = 1$, and $f(t) \to 0$ as $t \to +\infty$). Thus (x, t) is (weakly) preferred to ($\mathbf{y}$, $\mathbf{s}$) if and only if $\Sigma_i u(x_i)f(t_i) \geq \Sigma_i u(y_i)f(s_i)$. In this section, the comparative criterion is defined on *simple prospects*, consisting of single dated outcomes (written as (x, t), etc.).[5] In Section IV, I give an interpretation in the context of bipolar prospects—either cost–benefit or benefit–cost sequences.

[4] Note that the derivative, $d/dt\{-f^{*\prime}(t)/f^*(t)\}$, is independent of e^{ct}.

[5] Fishburn and Rubinstein (1982) provide an axiomatic derivation of the separable representation $u(x)f(t)$ using simple prospects only.

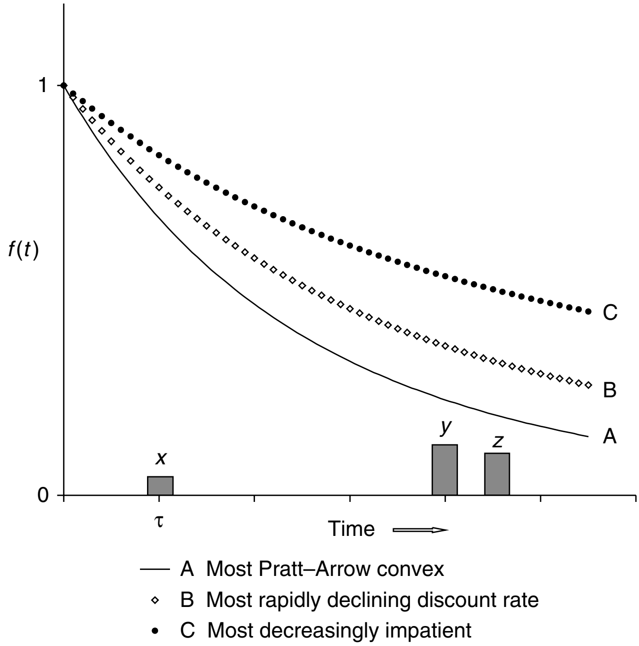

Fig. 2. Discount functions that exemplify three distinct criteria for assessing deviance from compound discounting. All three functions are actually-hyperbolic, $f(t) = (1 + \alpha t)^{-\beta/\alpha}$ (A: $\alpha = 1$, $\beta = 8$; B: $\alpha = 2$, $\beta = 6$; C: $\alpha = 3$, $\beta = 4$, plotted over [0, 0.3]). The vertical bars represent three mutually exclusive outcomes. A person can choose z immediately, or wait until time τ, at which point he can choose between x and y. In this two-stage decision problem, a choice of the dominated outcome z *is* compatible with discount function C (for some utilities $u(x) < u(z) < u(y)$) but *is not* compatible with discount functions A or B. In this sense, C promotes more violations of dominance

Definition 1. $\geq$ exhibits *decreasing impatience* (or *DI*) if for any $\sigma > 0$, $y > x > 0$, $(x, t) \sim (y, s)$ implies $(y, s + \sigma) \geq (x, t + \sigma)$ (*strict decreasing impatience* if $(y, s + \sigma) > (x, t + \sigma)$).

DI constrains violations of stationarity to favor the larger benefit (y) if both benefits are contemplated from a more distant vantage point. To discriminate between greater and smaller violations of stationarity, one can postpone the larger benefit by a greater time interval, from s to $s + \rho + \sigma$ (instead of $s + \sigma$), and then check whether a DI preference reversal is still sustained. This motivates the comparative definition:

Definition 2. $\geq$ exhibits *more decreasing impatience* than $\geq^*$ if for any intervals $0 \leq t < s$, ρ, σ, and outcomes $0 < x < y$, $0 < x' < y'$, $(x, t) \sim (y, s)$, $(x, t + \sigma) \sim (y, s + \rho + \sigma)$, and $(x', t) \sim^* (y', s)$ imply $(x', t + \sigma) \geq^* (y', s + \rho + \sigma)$.

We now prove an equivalence between this comparative property of preferences and the selection of dominated outcomes. Formally, a prospect (x, t) *dominates* (y, s) or $(x, t) \gg (y, s)$ iff $(x, t) \geq (y, t)$ and $(x, t) > (x, s)$, which, given our assumptions, means that (x, t) gives at least as good an outcome as (y, s) at an earlier point in time.

Figure 4 displays examples of simple, two-stage decision problems, where a person may choose a dominated prospect. In each panel, there is an immediate decision—"today"—between obtaining prospect (x, r), or passing to a second-stage decision at time τ—"tomorrow"—between prospects (x, t) and (y, s) (with t, s measured relative to tomorrow's decision point, at τ). We say that $\geq$ allows for *sophisticated two-stage violations of dominance* at times (t, s, r, τ) if $\geq$ is DI and there exist outcomes x, y, such that:

$$\begin{aligned} \text{Today } (0): &\quad (x, t+\tau) \gg (x, r) \geq (y, s+\tau); \\ \text{Tomorrow } (\tau): &\quad (x, t) < (y, s). \end{aligned}$$

If preferences are DI, then the pattern $(x, t) < (y, s)$, $(x, t+\tau) > (y, s+\tau)$, implies that $u(x) > u(y)$ (Figure 3, top panel). Notice, also, that there is no loss of generality in keeping the same outcome in (x, t) and (x, r), because if the pattern of preferences $(x, t+\tau) \gg (z, r) \geq (y, s+\tau)$, $(x, t) < (y, s)$ holds for some $z \neq x$, then $(x, t+\tau) \gg (z, r)$ implies $u(z) \leq u(x)$ and hence the pattern will hold for $z = x$. The person anticipates a choice of the inferior outcome tomorrow, and hence precommits to the superior outcome today, even though it means an extra delay relative to choosing y tomorrow.

Similarly, we say that $\geq$ allows *naïve two-stage violations of dominance* at times (t, s, r, τ) if $\geq$ is DI and there exist outcomes x, y, such that:

$$\begin{aligned} \text{Today } (0): &\quad (y, s+\tau) \geq (x, r) \gg (x, t+\tau); \\ \text{Tomorrow } (\tau): &\quad (y, s) < (x, t). \end{aligned}$$

This time, if preferences are DI, then the pattern $(x, t) > (y, s)$, $(x, t+\tau) \leq (y, s+\tau)$, implies that outcome y is better than outcome x (Figure 3, bottom panel). Today, the naïve person prefers y, but tomorrow his preferences change, and he goes for the inferior outcome, x.[6]

[6] Alternatively, the dominance criterion could have been defined by only specifying the two time periods in the bottom branch of Figure 3, and then asking whether there exist outcomes x, y, z, *and a date* r such that the dominance-violating preference pattern holds. By that criterion, however, a dominance-violating combination would always exist, as long as preferences are strictly DI. Hence, the criterion would fail to discriminate among strictly DI functions. See also Proposition 5 in O'Donoghue and Rabin (1999a) which shows that any degree of present bias in quasi-hyperbolic preferences leads to some dominance violations.

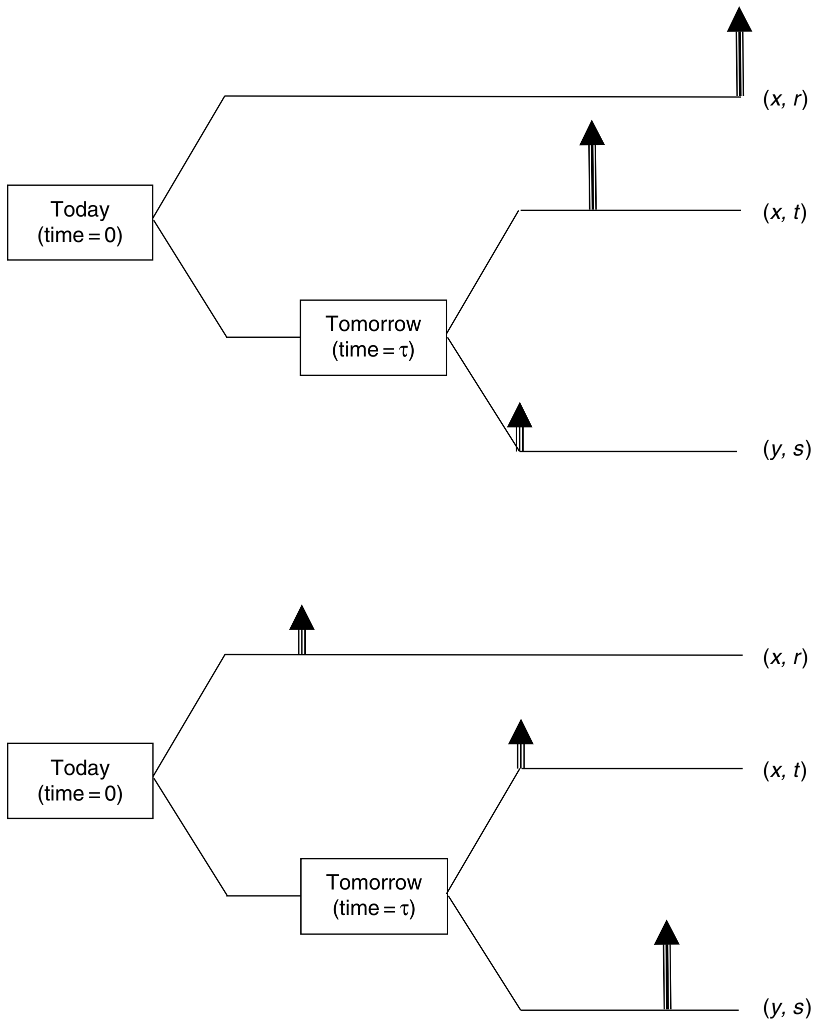

Fig. 3. Two-stage decision problems leading to dominated choices. Arrow length indicates magnitude of outcomes. Prospects are dated relative to last decision point, i.e., (x, t) arrives t units after "tomorrow" or $t + \tau$ after "today". The top panel displays a problem where a sophisticate, anticipating choice of (y, s) tomorrow, chooses the dominated prospect (x, r) today. The bottom panel displays a problem where a naïf expects to choose (y, s) tomorrow, but actually chooses the dominated prospect (x, t)

Definition 3. $\geq$ allows more sophisticated [naïve] violations of dominance than $\geq^*$ if for any t, s, r, τ, if $\geq^*$ allows sophisticated [naïve] two-stage violation of dominance at (t, s, r, τ) then $\geq$ allows sophisticated [naïve] two-stage violation of dominance at (t, s, r, τ).

For either type of agent, a time discount function leads to more dominated choices if and only if the log of the discount function is more convex:[7]

Proposition 1. *Let $\geq$ and $\geq^*$ be two preference orders represented by $u(x)$, $f(t)$, and $u^*(x), f^*(t)$, respectively. Then the following conditions are equivalent*:
(*a*) $\geq$ *exhibits more decreasing impatience than* $\geq^*$;
(*b*) $\geq$ *allows more sophisticated violations of dominance than* $\geq^*$;
(*c*) $\geq$ *allows more naïve violations of dominance than* $\geq^*$;
(*d*) $\ln f(f^{*-1}(e^z))$ *is convex in* $z \leq 0$.

Proof: See Appendix.

Part (d) states that the graph of $\ln f$ is more convex than the graph of $\ln f^*$, by the Pratt–Arrow definition of comparative convexity; see Pratt (1964). We can use Proposition 1 to efficiently define DI preferences. If $\geq^*$ are represented by a compound discount function, $f^*(t) = \delta^t$, then $f^{*-1}(e^z) = z/\ln \delta$. Hence, $\ln f(f^{*-1}(e^z)) = \ln f(z/\ln \delta)$, from which follows (as $z \leq 0$, $\delta < 1$ or $t = z/\ln \delta \geq 0$):

Corollary 1. *Let $\geq$ be a preference order represented by $u(x), f(t)$. Then the following conditions are equivalent*:
(*a*) $\geq$ *exhibits decreasing impatience* (*Definition 1*);
(*b*) $\ln f(t)$ *is convex in* $t \geq 0$.

IV. Procrastination on Investments and Promotion of Consumption

The schemata in Figure 3 can be modified so as to highlight alternative psychological interpretations of the same basic result. We can recast the choice between simple prospects as a choice between a mixed, gain–loss prospect $(x, t; y, s)$, and a null, status-quo outcome, (0). If the initial outcome is negative, the mixed prospect becomes a generic investment opportunity, such as a savings action, or some unpleasant chore. If the initial outcome is positive, the prospect represents a generic consumption opportunity—an indulgence with a delayed cost. In this section, we look at timing decisions for these prospects, whether to do them today, tomorrow, or not at all.

We start with investment opportunities, so that $x < 0 < y$. The situation is displayed in Figure 4, top panel. The person has a choice between doing "a chore" now or postponing the decision until tomorrow, when again there will be a choice between doing the chore or not doing it at all (option (0) in the figure). The timing of the benefit does not change, hence doing the chore

[7] Proposition 3 in Prelec (1989) states an equivalence between (d) and the selection of dominated prospects, but the formulation is somewhat different from any condition given here.

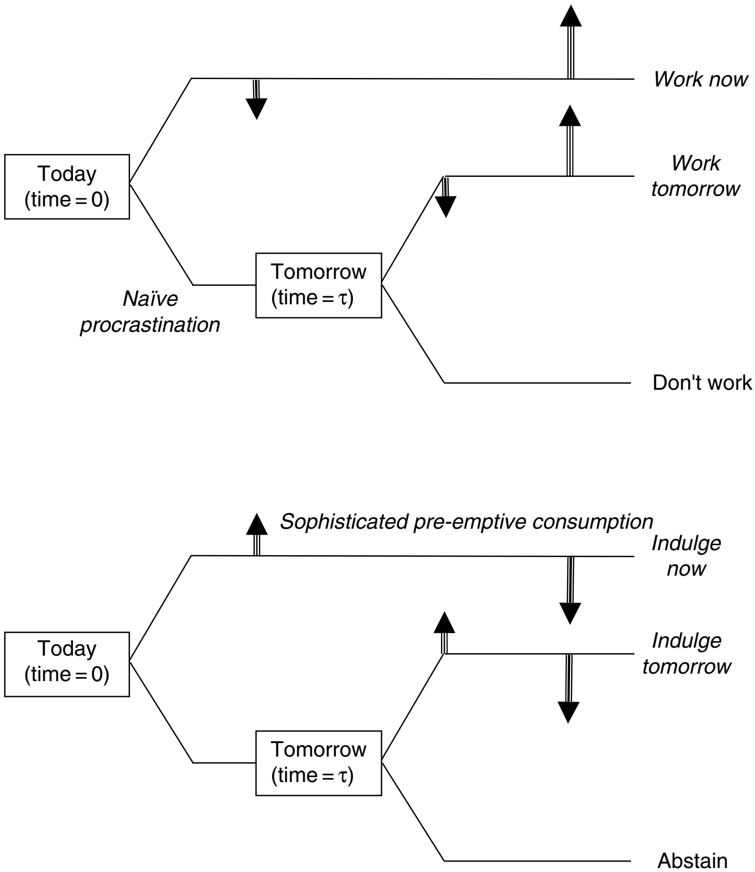

Fig. 4. Reinterpretation of the bottom schema in Figure 3 in terms of generic investment opportunities (top panel) and generic consumption opportunities (bottom panel). In the top panel, the decision is whether to work today or tomorrow, or not work at all. A naïf decides to work tomorrow (procrastinates) but fails to carry out this intention. In the bottom panel, abstaining is the best option from today's vantage point. However, a sophisticate knows that he will fail to abstain tomorrow and pre-emotively chooses to indulge today. The analysis of consumption opportunities is analogous

tomorrow is dominating. However, the negative initial outcome is closer to the decision point tomorrow (e.g. because there is less time remaining and "work must start right away").

We say that $\geq$ *allows procrastination* at times (t, s, r, τ), if $\geq$ is DI and there exist outcomes $x < 0 < y$, such that:

$$\text{Today (0):} \quad (x, t + \tau; y, s + \tau) > (x, r; y, s + \tau) \geq (0);$$
$$\text{Tomorrow } (\tau): \quad (0) > (x, t; y, s).$$

This is procrastination. A person prefers to do the chore immediately $(x, r; y, s+\tau)$ rather than not at all (0), but postponing the chore $(x, t+\tau; y, s+\tau)$ is even better. However, when tomorrow arrives, the chore will not get done. Comparing two naïve individuals, with preferences $\geq$ and $\geq^*$, we say that,

Definition 4. $\geq$ allows more procrastination on investment opportunities than $\geq^*$ if for any t, s, r, τ, if $\geq^*$ allows procrastination at (t, s, r, τ) then $\geq$ allows procrastination at (t, s, r, τ).

We say that $\geq$ *allows pre-emptive consumption* at times (t, s, r, τ), if $\geq$ is DI and there exist outcomes $x > 0 > y$, such that:

$$\begin{aligned} \text{Today } (0)&: \quad (0) \geq (x, r; y, s+\tau) > (x, t+\tau; y, s+\tau); \\ \text{Tomorrow } (\tau)&: \quad (x, t; y, s) > (0). \end{aligned}$$

The best option from today's vantage point is to abstain (0), and the worst is to consume tomorrow $(x, t+\tau; y, s+\tau)$. The naïve agent falsely believes that she will abstain tomorrow, and hence gets stuck with the worst prospect. A sophisticated agent would correctly anticipate this choice, and would therefore choose to consume today $(x, r; y, s+\tau)$ even though this is inferior to abstention (from today's vantage point). Her logic is—I will certainly give in tomorrow, so why not give in today? The formal definition is parallel to the previous one.

Definition 5. $\geq$ allows more pre-emptive consumption than $\geq^*$ if for any t, s, r, τ, if $\geq^*$ allows pre-emptive consumption at (t, s, r, τ) then $\geq$ allows pre-emptive consumption at (t, s, r, τ).

Definitions 4 and 5 are both derived from the second decision diagram in Figure 3 (which corresponds to Definition 2). Definition 5 subtracts outcome (y, s) from all three options in the diagram; hence, the lower path leading to outcome (y, s) in Figure 3 (bottom), becomes the abstain path in Figure 4 (bottom), etc. Definition 4 merely reverses the sign on all options in Definition 5. Hence, in a separable model, Definitions 4 and 5 are both equivalent to Definition 2. Summarizing,

Corollary 2. *Let $\geq$ and $\geq^*$ be two preference orders represented by $u(x)$, $f(t)$, and $u^*(x)$, $f^*(t)$, respectively. Then the following conditions are equivalent*:
(*a*) *$\geq$ exhibits more decreasing impatience than $\geq^*$*;
(*b*) *$\geq$ allows more naïve procrastination than $\geq^*$*;
(*c*) *$\geq$ allows more sophisticated pre-emption than $\geq^*$*;
(*d*) *$\ln f(f^{*-1}(e^z))$ is convex in $z \leq 0$.*

V. Non-stationarity "in the Small"

The definition of more or less DI in Proposition 1 may also be applied to small time intervals, to assess whether one or another preference order deviates more from stationarity near a point in time. With twice differentiable discount functions, there is an interesting interpretation of DI as a gap between momentary impatience and momentary time preference. Because it is customary to identify impatience with time preference, it may seem unusual to speak of any gap here. However, the core meaning of impatience is a preference for something to happen sooner rather than later, which is conceptually distinct from a preference for whether something should happen at all. A person may dislike a temporal prospect, but at the same time prefer to have it happen sooner if it cannot be avoided. This is precisely the situation described in Figure 4 (bottom panel).

We first define an *impatience function*,[8]

$$g(t) = \frac{f'(t)}{f'(0)},$$

with the formal properties of a discount function, $g(0) = 1$, $g'(t) < 0$, which determines—by calculation of present value, $\Sigma_i u(x_i) g(t_i)$—whether a person would prefer to accelerate (x, t) by at least a small time interval (because $d/dt\{\Sigma_i u(x_i) f(t_i)\} > 0 \leftrightarrow \Sigma_i u(x_i) g(t_i) > 0$).

With compound discounting $f(t) = g(t) = \delta^t$, and the impatience function is the same as the discount function. However, for DI functions, $g(t) \leq f(t)$. Hence, early outcomes will have relatively more impact on timing decisions—decisions whether to accelerate or delay a prospect—than on decisions whether to accept or reject prospects, when no adjustment in timing is possible. For instance, investment prospects, with an initial cost and delayed benefits, may have positive present value according to $f(t)$ and a negative present value according to $g(t)$. They might look good on a take-it-or-leave-it basis, but would look even better if postponed (e.g., the present value of the cost–benefit prospect displayed in Figure 1).

The difference between the impatience rate and the rate of time preference measures local non-stationarity of preferences, in the DI direction:

Proposition 2. Let $\geq$ *and* $\geq^*$ *be two preference orders represented by* $u(x)$, $f(t)$, *and* $u^*(x)$, $f^*(t)$, *with* $f(t)$ *and* $f^*(t)$ *twice differentiable, yielding*

[8] In Prelec (1989), I refer to this as *timing impatience*, and show that more timing impatience is equivalent to greater preference for delaying investment prospects (cost followed by benefit as in Proposition 4, p. 23). It is also equivalent to greater Pratt–Arrow convexity of the original discount function, $f(t)$.

impatience functions $g(t) = -f'(t)/f'(0)$, $g^*(t) = -g^{*\prime}(t)/g^{*\prime}(0)$. *Then the following are equivalent*:

(*a*) $\geq$ *exhibits more decreasing impatience than* $\geq^*$;

(*b*) *the difference between the impatience rate and the rate of time preference is greater with f than with f**:

$$\left(-\frac{g'(t)}{g(t)}\right) - \left(-\frac{f'(t)}{f(t)}\right) \geq \left(-\frac{g^{*\prime}(t)}{g^*(t)}\right) - \left(-\frac{f^{*\prime}(t)}{f^*(t)}\right), \quad \textit{for all } t > 0,$$

or, in words,

Decreasing impatience = Impatience − Time preference

Proof: See Appendix.

The terms in parentheses in 2(b) are momentary rates, of impatience and of time preference. For example, with the actually-hyperbolic, $f(t) = (1 + \alpha t)^{-\beta/\alpha}$, the impatience function is $g(t) = (1 + \alpha t)^{-\beta/\alpha - 1} = f(t)/(1 + \alpha t)$. Hence, in terms of rates, the decomposition is:

$$\frac{\alpha}{1 + \alpha t} = \frac{\alpha + \beta}{1 + \alpha t} - \frac{\beta}{1 + \alpha t}.$$

VI. Applications

Proposition 1 can clarify which parameters in a functional form control the DI aspect of preference. Note, first, that:

Corollary 3. *Two discount functions* $f_1(t)$ *and* $f_2(t)$ *represent equally decreasingly impatient preferences if and only if they are related by the power transformation,* $f^*(t) = f(t)^c$.

Proof: ($\Rightarrow$) $f^*(t) = f(t)^c$ implies $f^{*-1}(z) = f^{-1}(z^{1/c})$, which in turn implies that: $\ln f(f^{*-1}(e^z)) = \ln f(f^{-1}(e^{z/c})) = z/c$, which is convex. Hence f is more DI than f^* and, similarly, f^* is more DI than f, proving that f and f^* are equally DI. ($\Leftarrow$) If f and f^* are equally DI then $\ln f^* = c \ln f$ (otherwise $\ln f$ would not be a convex transform of $\ln f^*$, or vice versa), i.e., $f^*(t) = f(t)^c$. ■

In particular, this means that time preference (the c-parameter) can be adjusted independently within a given DI class. For example, two actually-hyperbolic functions, $(1 + \alpha t)^{-\beta/\alpha}$ and $(1 + \alpha^* t)^{-\beta^*/\alpha^*}$, are related by a power transformation iff $\alpha = \alpha^*$, which means that the α-parameter controls DI, while the β-parameter controls time preference.

With the quasi-hyperbolic form, the situation is a bit more complex, as both parameters are involved. If $f(t) = \beta\delta^t$, $t > 0$, $f(0) = 1$, and $f^*(t) = \beta^*\delta^{*t}$, $t > 0$, $f^*(0) = 1$, then $f^*(t) = f(t)^c$ iff $\ln \beta / \ln \delta = \ln \beta^* / \ln \delta^*$. Therefore, the ratio $\tau = \ln \beta / \ln \delta$ captures the DI aspect of preferences, with $\beta = 1$ yielding $\tau = 0$. Because $\beta = \delta^\tau$, we can think of τ as a "virtual time interval" associated with the immediate present in the context of an underlying discount rate δ. Although β is nominally the "present-bias" parameter, a high discount rate dilutes the present-bias, as it were. Writing the quasi-hyperbolic as $f(t) = \delta^{\tau + t}$, separates time preference (δ) from decreasing impatience (τ).

Earlier, we had briefly considered the drop in the discount rate as a possible criterion for assessing non-stationary time preference (exemplified by function B in Figure 1). With that criterion, two discount functions are equivalent if $f^*(t) = e^{ct} f(t)$, which, in the context of quasi-hyperbolic preferences, would imply $\beta = \beta^*$. This does not seem quite right, if interpreted as indicating equal levels of deviance from stationarity. One would think, for example, that a person with $\beta^* = \frac{1}{2}$, $\delta^* = 1$ is more deviant than a person with $\beta = \delta = \frac{1}{2}$.

In some applications it is natural to consider mixtures of discount functions, for example when looking at time preferences of multi-agent units, whose members have different rates of time preferences or when a single agent has a stochastic discount function; see Bernheim (2000), Ebert and Prelec (2003) and Harris and Laibson (2003). The general result is that a mixture of discount functions, drawn from the same DI class, but differing in the rate of time preference, will be more DI than any of the constituents:

Corollary 4. *If different discount functions $f_1(t)$ and $f_2(t)$ represent equally DI preferences, then the mixture of the two, $f(t) = \lambda f_1(t) + (1 - \lambda) f_2(t)$, represents more DI preferences.*

Proof: If $f_1(t)$ and $f_2(t)$ are different but represent equally DI preferences, then by Corollary 3 we can write, $f_1(t) = f_2(t)^c$, for some $c > 0$, $c \neq 1$. By Proposition 1(d), we need to check convexity of the function $\ln f(f_1^{-1}(e^z)) = \ln(\lambda f_1(f_1^{-1}(e^z)) + (1-\lambda) f_2(f_1^{-1}(e^z))) = \ln(\lambda e^z + (1-\lambda) e^{z/c})$, which is indeed convex in $z \leq 0$. Hence, by Proposition 1, it follows that $f(t)$ represents more DI preferences than $f_1(t)$ (or $f_2(t)$—the argument is identical). ■

In effect, the function with the higher rate of time preference controls the shape of the mixed function in the early time periods, while the function with the lower rate of time preference takes over for the more distant periods. The result can also be used to compare mixtures: a 50–50 mixture will be more DI than a 25–75 mixture, and so forth.

Finally, Proposition 1 directs attention to new functional forms. The general method is to formulate conditions under which preferences might be DI-invariant, and then derive the corresponding discount function. A promising example here is $f(t) = \exp\{-\beta t^{\alpha}\}$, which is convenient to estimate, and provides a remarkably good fit to the data; see Ebert and Prelec (2003). The argument for this function is similar to arguments for the constant-relative risk-aversion utility function, which likewise rest on an invariance with respect to change in scale. In the temporal setting, we assume that changing the time scale for the discount function does not affect DI. This means that to resolve whether the difference between waiting 1 or 2 time units is psychologically greater (requiring more compensation) than the difference between waiting 5 or 7 time units, one does not need to know whether the numbers 1, 2, 5, 7 refer to days, weeks or years. This is reasonable if hyperbolic discounting is an artifact of relative similarity judgments; see Rubinstein (2003). Such judgments presumably depend on the similarity of numbers and not on the placement of the decimal point. The derivation then proceeds like this: by hypothesis, the original discount function $f(t)$ is equally DI as the function $f(\lambda t)$, which represents a change in time scale. By Corollary 3, this implies a functional equation $f(\lambda t) = f(t)^{c}$, whose solution is $f(t) = \exp\{-\beta t^{\alpha}\}$.

VII. Conclusion

Here I have presented the case in favor of treating decreasing impatience as the core property that is parametrically expressed by hyperbolic and quasi-hyperbolic discount functions. Decreasing impatience provides a natural criterion for assessing whether a set of time preferences represents a more or less severe departure from the stationarity axiom. The criterion is associated with a simple normative diagnostic—the selection of inefficient (dominated) outcomes in two-stage decision problems. In separable representations, it leads to a Pratt–Arrow classification of discount functions, with concise global and local interpretations of deviation from exponential discounting.

Although the similarity to Pratt–Arrow treatment of utility functions is reassuring, it is debatable whether any one measure will replicate the success of the renowned Pratt–Arrow coefficient. Formally, intertemporal decisions seem to have greater intrinsic complexity than decisions involving risk. The definitions set forth here involved two-outcome prospects, in two-stage dynamic settings. It is an open research question whether decreasing impatience, or some variant property, can provide a unified treatment of more elaborate prospects, in multi-stage settings.

Appendix. Proofs

Proposition 1

(a) $\Leftrightarrow$ (d). The proof follows the proof of Proposition 3 in Prelec (1998), which asserts a similar equivalence in the context of separable representation of preferences over risky prospects, $v(x)w(p)$ (this, in turn, is similar to a proof by Wakker (1994) in the context of utility functions). Basically, the probability weighting function $w(p)$ in Prelec (1998) may be reinterpreted as a discount function $f(t) = w(e^{-t})$, and probabilities as time intervals, $t = -\ln p$.

Proof: (d) $\Rightarrow$ (a): The claim (d) that $\ln f(f^{*-1}(e^{z}))$ is convex in z for $z \leq 0$ is equivalent to the claim that $h(h^{*-1}(t))$ is convex in $t \geq 0$, for $h(t) = \ln f(t)$, $h^*(t) = \ln f^*(t)$. I will show that if (a) fails, then $h(h^{*-1}(t))$ cannot be convex.

Assume that (a) fails, and that there exist $x, y, x', y', s, t, \sigma, \rho$ such that $(x, t) \sim (y, s)$, $(x, t+\sigma) \sim (y, s+\rho+\sigma)$, and $(x', t) \sim^* (y', s)$ and $(x', t+\sigma) <^* (y', s+\rho+\sigma)$. From $(x, t) \sim (y, s)$, $(x, t+\sigma) \sim (y, s+\rho+\sigma)$, we have,

$$\frac{f(s+\rho+\sigma)}{f(t+\sigma)} = \frac{u(x)}{u(y)} = \frac{f(s)}{f(t)}. \tag{A1}$$

However, $(x', t) \sim^* (y', s)$ and $(x', t+\sigma) <^* (y', s+\rho+\sigma)$ imply,

$$\frac{f^*(s+\rho+\sigma)}{f^*(t+\sigma)} > \frac{u(x')}{u(y')} = \frac{f^*(s)}{f^*(t)}. \tag{A2}$$

Letting $h(t) = \ln f(t)$, $h^*(t) = \ln f^*(t)$, we can write (A1) as $\exp(h(s+\rho+\sigma))/\exp(h(t+\sigma)) = \exp(h(s))/\exp(h(t))$, or $h(s+\rho+\sigma) - h(t+\sigma) = h(s) - h(t)$. Likewise, from (A2), we have $\exp(h^*(s+\rho+\sigma))/\exp(h^*(t+\sigma)) > \exp(h^*(s))/\exp(h^*(t))$, or $h^*(s+\rho+\sigma) - h^*(t+\sigma) > h^*(s) - h^*(t)$. Because both h and h^* are decreasing, the equality $h(s+\rho+\sigma) - h(t+\sigma) = h(s) - h(t)$ and inequality $h^*(s+\rho+\sigma) - h^*(t+\sigma) > h^*(s) - h^*(t)$ are jointly incompatible with the claim that $h(t)$ is a convex transformation of $h^*(t)$, which is to say that $\ln f(t)$ is a convex transformation of $\ln f^*(t)$. This proves $\neg$ (a) $\Rightarrow$ $\neg$ (d), or (d) $\Rightarrow$ (a). ■

Proof: (a) $\Rightarrow$ (d): As before, we let $h(t) = \ln f(t)$, $h^*(t) = \ln f^*(t)$, and show now that for any $0 \leq t < s \leq t+\sigma < s+\sigma+\rho$, $h(t) - h(s) = h(t+\sigma) - h(s+\sigma+\rho)$ implies $h^*(t) - h^*(s) \leq h^*(t+\sigma) - h^*(s+\sigma+\rho)$, which proves that h is a convex transformation of h^* (as h and h^* are decreasing functions).

Choose any $0 \leq t < s \leq t+\sigma < s+\sigma+\rho$, such that $h(t) - h(s) = h(t+\sigma) - h(s+\sigma+\rho)$. By definition of h as $\ln f$, this implies, $\ln f(t) - \ln f(s) = \ln f(t+\sigma) - \ln f(s+\sigma+\rho))$, or $f(t)/f(s) = f(t+\sigma)/f(s+\sigma+\rho)$. Because $u(x)$ is a continuous ratio scale with $u(0) = 0$, we can choose $0 < x < y$ such that,

$$\frac{f(s+\rho+\sigma)}{f(t+\sigma)} = \frac{u(x)}{u(y)} = \frac{f(s)}{f(t)}.$$

Hence,

$$f(t)u(x) = f(s)u(y), \quad f(t+\sigma)u(x) = f(s+\rho+\sigma)u(y)$$

and

$$(x,t) \sim (y,s), \quad (x,t+\sigma) \sim (y,s+\rho+\sigma).$$

Because u^* is a continuous ratio scale with $u^*(0) = 0$, we can choose x', y', such that,

$$\frac{u^*(x')}{u^*(y')} = \frac{f^*(s)}{f^*(t)},$$

which implies $(x', t) \sim^* (y', s)$. But, given (a), the pattern $(x, t) \sim (y, s)$, $(x, t+\sigma) \sim (y, s+\rho+\sigma)$, $(x', t) \sim^* (y', s)$ implies: $(x', t+\sigma) \geq^* (y', s+\rho+\sigma)$, or:

$$\frac{u^*(x')}{u^*(y')} \geq \frac{f^*(s+\rho+\sigma)}{f^*(t+\sigma)}.$$

Hence, $f^*(t)/f^*(s) \leq f^*(s+\rho+\sigma)/f^*(s+\rho+\sigma)$, i.e., $\ln f^*(t) - \ln f^*(s) \leq \ln f^*(s+\rho+\sigma) - \ln f^*(s+\rho+\sigma)$, i.e., $h^*(t) - h^*(s) \leq h^*(s+\rho+\sigma) - h^*(s+\rho+\sigma)$. We have therefore shown that for any $0 \leq t < s \leq t+\sigma < s+\sigma+\rho$, $h(t) - h(s) = h(t+\sigma) - h(s+\sigma+\rho)$ implies $h^*(t) - h^*(s) \leq h^*(t+\sigma) - h^*(s+\sigma+\rho)$, proving that h is a convex transformation of h^* (as h and h^* are decreasing functions). ■

Proof: (a) ⇔ (b): Note first that (a) is equivalent to the following constraint on the discount functions representing $\geq$ and $\geq^*$:

$$\frac{f(s)}{f(t)} = \frac{f(s+\rho+\sigma)}{f(t+\sigma)} \Rightarrow \frac{f^*(s)}{f^*(t)} \geq \frac{f^*(s+\rho+\sigma)}{f^*(t+\sigma)}, \quad \text{for all } s > t, \sigma, \rho. \tag{A3}$$

Equation (A3) may be derived directly from the preference conditions, $(x, t) \sim (y, s)$, $(x, t+\sigma) \sim (y, s+\rho+\sigma)$, $(x', t) \sim^* (y', s)$ implies: $(x', t+\sigma) \geq^* (y', s+\rho+\sigma)$. Conversely, if (A3) fails, then by continuity of the utility function at zero we can find outcomes x, y that lead to a violation of these preference conditions.

We now show that (b) is equivalent to (A3). Observe that $\geq$ allows for two-stage violations of dominance by a sophisticated agent at times (t, s, r, τ) iff: $t+\tau < r$, $s < t$, and:

$$\frac{f(r)}{f(s+\tau)} > \frac{f(t)}{f(s)}.$$

if $t+\tau \geq r$ then we could not have $(x, t+\tau) \gg (x, r)$); if $s \geq t$, then the required pattern, $(x, t) < (y, s)$, $(x, t+\tau) > (y, s+\tau)$ would be inconsistent with DI. If the above inequality holds, then by continuity of the utility function at zero, one can find outcomes x, y, such that,

$$\frac{f(r)}{f(s+\tau)} \geq \frac{u(y)}{u(x)} > \frac{f(t)}{f(s)},$$

implying $(x, r) \geq (y, s+\tau)$ and $(y, s) > (x, t)$, and because $t+\tau < r$, we also have $(x, t+\tau) \gg (x, r)$. (b) is therefore equivalent to the claim that,

$$\frac{f^*(r)}{f^*(s+\tau)} > \frac{f^*(t)}{f^*(s)} \Rightarrow \frac{f(r)}{f(s+\tau)} > \frac{f(t)}{f(s)}, \quad \text{for } t+\tau < r, s < t,$$

or, equivalently,

$$\frac{f(t)}{f(s)} \geq \frac{f(r)}{f(s+\tau)} \Rightarrow \frac{f^*(t)}{f^*(s)} \geq \frac{f^*(r)}{f^*(s+\tau)}, \quad \text{for } t+\tau < r, s < t.$$

Because $t+\tau < r$, we can let $\sigma = r - t - \tau > 0$, and, letting $\rho = \tau$, we can write the above implication as,

$$\frac{f(t)}{f(s)} \geq \frac{f(t+\rho+\sigma)}{f(s+\rho)} \Rightarrow \frac{f^*(t)}{f^*(s)} \geq \frac{f^*(t+\rho+\sigma)}{f^*(s+\rho)}, \quad \text{for all } s < t, \sigma, \rho. \tag{A4}$$

This is equivalent to (A3), given continuous f, f^*, and given that $s > t$ in (A3) and $s < t$ in (A4) (i.e., t, s in (A4) stand for s, t in (A3)). ■

Proof: (a) ⇔ (c): The main step again is to use the parameters s, t, ρ, σ, in (a) to identify a two-stage decision problem, with $r = t+\rho$, $\tau = \rho+\sigma$. Observe that $\geq$ will allow two-stage violations of dominance by a naïve agent at times (t, s, r, τ) iff,

$$\frac{f(s+\tau)}{f(r)} > \frac{f(s)}{f(t)}, \quad \text{for } t < r < t+\tau, s > t.$$

(If $t \geq r$, then we could not have: $(x, t) > (x, r)$ and hence we could not have $(x, t) > (y, s) \gg (y, s+\tau) \geq (x, r)$; if $r \geq \tau + t$, we could not have $(x, r) \gg (x, t+\tau)$; if $s \leq t$, we could not have DI and $(x, t) > (y, s)$ and $(x, t+\tau) \leq (y, s+\tau)$.) If the inequality holds, then by continuity of the utility function at zero, one can find outcomes x, y, such that,

$$\frac{f(s+\tau)}{f(r)} \geq \frac{u(x)}{u(y)} > \frac{f(s)}{f(t)},$$

implying $(s+\tau, y) \geq (x, r)$, $(x, t) > (y, s)$. Because $r < t+\tau$, we have $(x, r) \gg (x, t+\tau)$. (c) is therefore equivalent to the claim that,

$$\frac{f^*(s+\tau)}{f^*(r)} > \frac{f^*(s)}{f^*(t)} \Rightarrow \frac{f(s+\tau)}{f(r)} > \frac{f(s)}{f(t)}, \quad \text{for all } t < r < t+\tau.$$

Because $r > t$, we can let $r = t+\rho$, for $\rho > 0$, and because $r < t+\tau$, can write let $r+\sigma = t+\tau$, for $\sigma > 0$, which gives $\tau = r+\sigma-t = (t+\rho)+\sigma-t = \rho+\sigma$. Therefore, the implication equivalent to (c) may be stated as,

$$\frac{f^*(s+\rho+\sigma)}{f^*(t+\rho)} > \frac{f^*(s)}{f^*(t)} \Rightarrow \frac{f(s+\rho+\sigma)}{f(t+\rho)} > \frac{f(s)}{f(t)}, \quad s > t, \rho, \sigma,$$

i.e., (A5)

$$\frac{f(s)}{f(t)} \geq \frac{f(s+\rho+\sigma)}{f(t+\rho)} \Rightarrow \frac{f^*(s)}{f^*(t)} \geq \frac{f^*(s+\rho+\sigma)}{f^*(t+\rho)}, \quad s > t, \rho, \sigma,$$

which is equivalent to (A4), hence to (b) and (a) (note that $s > t$ in (A5), and $s < t$ in (A4)). ■

Proposition 2

By Proposition 1, a function is more DI iff $h = \ln f$ is more convex than $h^* = \ln f^*$, which is to say if $-\ln f$ is more concave than $-\ln f^*$. For twice differentiable increasing functions, this means that the Pratt–Arrow coefficient in Pratt (1964) is greater for $-\ln f$ than with $-\ln f^*$, i.e., that: $-(-\ln f)''/(-\ln f)' \geq -(-\ln f^*)''/(-\ln f^*)'$, or $(\ln f)''/(\ln f)' \leq (\ln f^*)''/(\ln f^*)'$. However, $(\ln f)''/(\ln f)' = f''/f' - f'/f = g'/g - f'/f = (-f'/f) - (-g'/g)$. Hence, f is more DI than f^* iff:

$$-\frac{g'(t)}{g(t)} - \left(-\frac{f'(t)}{f(t)}\right) \geq -\frac{g^{*\prime}(t)}{g^*(t)} - \left(-\frac{f^{*\prime}(t)}{f^*(t)}\right), \quad \text{at all } t \geq 0.$$

References

Ainslie, G. (1975), Specious Reward—Behavioral Theory of Impulsiveness and Impulse Control, *Psychological Bulletin 82* (4), 463–496.

Ainslie, G. (1982), A Behavioral Economic-approach to the Defense-mechanisms—Freud's Energy Theory Revisited, *Social Science Information sur les sciences sociales 21* (6), 735–779.

Ainslie, G. (1984), Behavioral Economics II—Motivated, Involuntary Behavior, *Social Science Information sur les sciences sociales 23* (1), 47–78.

Ainslie, G. (1989), Freud and Picoeconomics, *Behaviorism 17* (1), 11–19.

Ainslie, G. (1992), *Picoeconomics: The Strategic Interaction of Successive Motivational States within the Person*, Cambridge University Press, New York.

Benabou, R. and Tirole, J. (2002), Self-confidence and Personal Motivation, *Quarterly Journal of Economics 117* (3), 871–915.

Benzion, U., Rapoport, A. and Yagil, J. (1989), Discount Rates Inferred from Decisions—An Experimental-study, *Management Science 35* (3), 270–284.

Bernheim, B. D. (2000), Comment on "Family Bargaining and Retirement Behavior", by S. Lundberg, in H. Aaron (ed.), *Psychological Perspectives on Retirement*, Brookings Foundation and Russell Sage Foundation Press, Washington, DC, 273–281.

Brocas, I. and Carrillo, J. D. (2000), The Value of Information when Preferences are Dynamically Inconsistent, *European Economic Review 44* (4–6), 1104–1115.

Brocas, I. and Carrillo, J. D. (2001), Rush and Procrastination under Hyperbolic Discounting and Interdependent Activities, *Journal of Risk and Uncertainty 22* (2), 141–164.

Carrillo, J. and Mariotti, T. (2000), Strategic Ignorance as a Self-disciplining Device, *Review of Economic Studies 67*, 529–554.

Chapman, G. B. and Elstein, A. S. (1995), Valuing the Future—Temporal Discounting of Health and Money, *Medical Decision Making 15* (4), 373–386.

Chung, S. H. and Herrnstein, R. J. (1967), Choice and Delay of Reinforcement, *Journal of the Experimental Analysis of Behavior 10*, 67–74.

Ebert, J. J. E. and Prelec, D. (2003), Casual Discounting: The Impact of Mental Effort and Attention to the Future on Far and Near Future Valuations, working paper, University of Minnesota.

Fishburn, P. C. and Rubinstein, A. (1982), Time Preference, *International Economic Review 23*, 677–694.

Frederick, S., Loewenstein, G. and O'Donoghue, T. (2002), Time Discounting and Time Preference: A Critical Review, *Journal of Economic Literature 40* (2), 351–401.

Gruber, J. and Koszegi, B. (2001), Is Addiction "Rational"? Theory and Evidence, *Quarterly Journal of Economics 116* (4), 1261–1303.

Harris, C. and Laibson, D. (2001), Dynamic Choices of Hyperbolic Consumers, *Econometrica 69* (4), 935–957.

Harris, C. and Laibson, D. (2003), Hyperbolic Discounting and Consumption, in M. Dewatripont, L. P. Hansen and S. J. Turnovsky (eds.), *Advances in Economics and Econometrics: Theory and Applications*, *Eighth World Congress*, Cambridge University Press, Cambridge.

Herrnstein, R. J. (1981), Self-control as Response Strength, in C. M. Bradshaw, E. Szabadi and C. F. Lowe (eds.), *Quantification of Steady-state Operant Behavior*, Elsevier/North-Holland, Amsterdam.

Kirby, K. N. and Herrnstein, R. J. (1995), Preference Reversals Due to Myopic Discounting of Delayed Reward, *Psychological Science 6* (2), 83–89.

Kirby, K. N. and Marakovic, N. N. (1995), Modeling Myopic Decisions: Evidence for Hyperbolic Delay-discounting within Subjects and Amounts, *Organizational Behavior and Human Decision Processes 64* (1), 22–30.

Koopmans, T. C. (1960), Stationary Ordinal Utility and Impatience, *Econometrica 47*, 263–291.

Laibson, D. (1994), Essays in Hyperbolic Discounting, unpublished Ph.D. Dissertation, MIT, Cambridge, MA.

Laibson, D. (1997), Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics 112* (2), 443–477.

Loewenstein, G. and Prelec, D. (1992), Anomalies in Intertemporal Choice—Evidence and an Interpretation, *Quarterly Journal of Economics 107* (2), 573–597.

O'Donoghue, T. and Rabin, M. (1999a), Doing it Now or Later, *American Economic Review 89* (1), 103–124.

O'Donoghue, T. and Rabin, M. (1999b), Incentives for Procrastinators, *Quarterly Journal of Economics 114* (3), 769–816.

O'Donoghue, T. and Rabin, M. (2000), The Economics of Immediate Gratification, *Journal of Behavioral Decision Making 13* (2), 233–250.

O'Donoghue, T. and Rabin, M. (2001), Choice and Procrastination, *Quarterly Journal of Economics 116* (1), 121–160.

Phelps, E. S. and Pollak, R. (1968), On Second-best National Saving and Game-equilibrium Growth, *Review of Economic Studies 35*, 185–199.

Pratt, J. W. (1964), Risk Aversion in the Small and in the Large, *Econometrica 32*, 122–136.

Prelec, D. (1989), Decreasing Impatience: Definition and Consequences, Harvard Business School Working Paper no. 90-015.

Prelec, D. (1998), The Probability Weighting Function, *Econometrica 66* (3), 497–527.

Rubinstein, A. (2003), "Economics and Psychology"? The Case of Hyperbolic Discounting, *International Economic Review 44*, 1207–1216.
Strotz, R. H. (1955), Myopia and Inconsistency in Dynamic Utility Maximization, *Review of Economic Studies 23*, 165–180.
Thaler, R. (1981), Some Empirical Evidence on Dynamic Inconsistency, *Economics Letters 8*, 210–217.
Wakker, P. (1994), Separating Marginal Utility and Probabilistic Risk-aversion, *Theory and Decision 36* (1), 1–44.