

ANOMALIES IN INTERTEMPORAL CHOICE: EVIDENCE AND AN INTERPRETATION*

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Research on decision making under uncertainty has been strongly influenced by the documentation of numerous expected utility (EU) anomalies—behaviors that violate the expected utility axioms. The relative lack of progress on the closely related topic of intertemporal choice is partly due to the absence of an analogous set of discounted utility (DU) anomalies. We enumerate a set of DU anomalies analogous to the EU anomalies and propose a model that accounts for the anomalies, as well as other intertemporal choice phenomena incompatible with DU. We discuss implications for savings behavior, estimation of discount rates, and choice framing effects.

I. INTRODUCTION

Since its introduction by Samuelson in 1937, the discounted utility model (DU) has dominated economic analyses of intertemporal choice. In its most restrictive form, the model states that a sequence of consumption levels, (c_0, \dots, c_T) will be preferred to sequence (c'_0, \dots, c'_T) , if and only if,

$$(1) \quad \sum_{t=0}^T \delta^t u(c_t) > \sum_{t=0}^T \delta^t u(c'_t),$$

where $u(c)$ is a concave ratio scale utility function, and δ is the discount factor for one period. DU has been applied to such diverse topics as savings behavior, labor supply, security valuation, education decisions, and crime. It has provided a simple, powerful framework for analyzing a broad range of economic decisions with delayed consequences.

Yet, in spite of its widespread use, the DU model has not received substantial scrutiny—in marked contrast to the expected utility model for choice under uncertainty, which has been extensively criticized on empirical grounds and which has subsequently spawned a great number of variant models (reviewed, for example, by Weber and Camerer [1988]).

Our first aim in this paper is to remedy this imbalance by

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enumerating the anomalous empirical findings on time preference that have been reported so far. Taken together, they present a challenge to normative theory that is at least as serious as that posed by the much more familiar EU anomalies. Unlike the EU violations, which in many cases can only be demonstrated with a clever arrangement of multiple choice problems (e.g., the Allais paradox), the counterexamples to DU are simple, robust, and bear directly on central aspects of economic behavior. Our second aim is to construct (in Section III) a descriptive model of intertemporal choice that predicts the anomalous preference patterns. In formal structure the model is closely related to Kahneman and Tversky's "prospect theory" [1979], but the interpretation and shape of the component functions are different. The paper concludes with a discussion of some additional implications of the model for individual behavior and market outcomes.

II. FOUR ANOMALIES

In this section we present four common preference patterns that create difficulty for the discounted utility model.

1. The Common Difference Effect

Consider an individual who is indifferent between adding x units to consumption at time t and $y > x$ units at a later time t' , given a constant baseline consumption level (c) in all time periods:

$$(2) \quad u(c + x)\delta^t + u(c)\delta^{t'} = u(c)\delta^t + u(c + y)\delta^{t'}.$$

Dividing through by δ^t ,

$$(3) \quad u(c + x) - u(c) = (u(c + y) - u(c))\delta^{t'-t}$$

shows that preference between the two consumption adjustments depends only on the absolute time interval separating them, or $(t' - t)$ in the example above. This is the *stationarity* property, which plays a critical role in axiomatic derivations of the DU model [Koopmans, 1960; Fishburn and Rubinstein, 1982].

In practice, preferences between two delayed outcomes often switch when both delays are incremented by a given constant amount. An example of Thaler [1981] makes the point crisply: a person might prefer one apple today to two apples tomorrow, but at

the same time prefer two apples in 51 days to one apple in 50 days. We shall refer to this pattern as the *common difference effect*.¹

The common difference effect gives rise to dynamically inconsistent behavior, as noted first by Strotz [1956], and richly elaborated in the papers of the psychologist Ainslie [1975, 1985]. It also implies that discount rates should decrease as a function of the time delay over which they are estimated which has been observed in a number of studies, including one with real money outcomes [Horowitz, 1988].² See Figure VI for the results of Benzion et al. [1989], which are representative.

2. *The Absolute Magnitude Effect*

Empirical studies of time preference have also found that large dollar amounts suffer less proportional discounting than do small ones. Thaler [1981], for example, reported that subjects who were on average indifferent between receiving \$15 immediately and \$60 in a year, were also indifferent between an immediate \$250 and \$350 in a year, as well as between \$3000 now and \$4000 in a year. Similar results were obtained by Holcomb and Nelson [1989] with real money outcomes.

3. *The Gain-Loss Asymmetry*

A closely related finding is that losses are discounted at a lower rate than gains are. For example, subjects in a study by Loewenstein [1988c] were, on average, indifferent between receiving \$10 immediately and receiving \$21 in one year, and indifferent between losing \$10 immediately and losing \$15 in one year. The corresponding figures for \$100 were \$157 for gains and \$133 for losses. Even more dramatic loss-gain asymmetries were obtained by Thaler [1981], who estimated discount rates for gains that were three to ten times greater than those for losses. Several of his subjects actually exhibited negative discounting, in that they preferred an immediate loss over a delayed loss of equal value (also see Loewenstein [1987]).

1. The common difference effect is analogous to the common ratio effect in decision making under uncertainty [Kahneman and Tversky, 1979]. For a discussion of similarities and differences between the EU and DU axioms, see Prelec and Loewenstein [1991].

2. Horowitz [1988] used a second price sealed bit auction to estimate discount rates for \$50 "bonds" of varying maturity. Implicit discount rates were a declining function of time to bond maturity.

The magnitude and gain-loss effects are problematic for DU in two senses. First, the predictions that DU makes are sensitive to the baseline consumption profile, since the baseline level at a given time period directly controls the marginal utility of an extra unit of consumption. Experimental subjects represent a diversity of baseline levels of consumption, yet these choice patterns are consistent over a wide range of income (and hence consumption) levels. This pattern evokes the comments of Markowitz [1952] on the Friedman-Savage explanation for simultaneous gambling and insurance purchases. Friedman and Savage argued that simultaneous gambling and insurance could be explained by a doubly inflected utility function defined over levels of wealth. Markowitz pointed out that no single utility function defined over levels of wealth could explain why people at vastly different levels of wealth engage in both activities; a function that predicted simultaneous gambling and insuring for people at one wealth level would make counterintuitive predictions for people at other wealth levels.

Second, even the determinate predictions that DU yields, on the assumption that the baseline consumption level is constant across time periods, are not entirely consistent with the effects just described. Note first that the present value of a consumption change at time t , from c to $c + x$, can be measured in two ways, either by assessing the *equivalent* present value $q(x,t)$ defined implicitly by

$$(4) \quad u(c + q) + \delta^t u(c) = u(c) + \delta^t u(c + x),$$

or by assessing the *compensating* present value $p(x,t)$ that would exactly balance the change at time t :

$$(5) \quad u(c - p) + \delta^t u(c + x) = u(c) + \delta^t u(c).$$

(These are also referred to as the methods of *equivalent* and *compensating* variation.)

The gain-loss asymmetry is obtained by comparing the equivalent variation ratios (q/x) for positive and negative x . Here the DU model makes the correct qualitative prediction, as the following simple calculation shows:

$$(6) \quad q(x,t) = u^{-1}\{(1 - \delta^t)u(c) + \delta^t u(c + x)\} - c \quad (\text{solving from (4)}) \\ < (1 - \delta^t)c + \delta^t(c + x) - c \quad (\text{by concavity of } u(x)) \\ = \delta^t x.$$

Consequently, the ratio, $q(x,t)/x$, is smaller than δ^t for positive x and greater than δ^t for negative x , which is consistent with the observed greater relative discounting of gains.

The critical weakness of this explanation lies in the prediction it makes about the size of the gain-loss asymmetry at different absolute magnitudes. The normative explanation is driven by the global concavity of the utility function, which creates a gap (analogous to a risk premium) between time discounting and the pure rate of time preference. Since the utility function is approximately linear for small intervals ($c - x, c + x$), the gain-loss asymmetry should disappear for small x . Indeed, in the limit as x goes to zero (from either side), the predicted devaluation ratio q/x will approach the discount factor δ^t , for both gains and losses. In practice, however, we observe the exact opposite, with the gain-loss asymmetry being most pronounced for small outcomes [Thaler, 1981; Benzion et al., 1989].

With regard to the magnitude effect the DU predictions hinge partly on the method of elicitation. When present values are assessed by the equivalent variation method, DU contradicts the magnitude effect. For compensating variation, DU predicts the effect when x is negative, but predicts the exact opposite (i.e., smaller discounting of *small* amounts) for positive x . We now derive this last result as an illustration; the argument in the other cases is similar.

Suppose that p is the most one would be willing to pay now in order to receive $x > 0$ at time t , as in equation (5), and consider what happens as both p and x are increased by a common factor, $\alpha > 1$:

$$\begin{aligned}
 (7) \quad \frac{\partial}{\partial \alpha} \Big|_{\alpha=1} & \left\{ u(c - \alpha p) + \delta^t u(c + \alpha x) - (u(c) + \delta^t u(c)) \right\} \\
 & = -pu'(c - p) + \delta^t xu'(c + x) \quad (\text{from(5)}) \\
 & > 0 \quad (\text{if the magnitude effect holds}).
 \end{aligned}$$

After substituting for δ^t from (5), this inequality reduces to

$$(8) \quad pu'(c - p)(u(c + x) - u(c)) < xu'(c + x)((u(c) - u(c - p))).$$

But, since $u(c)$ is concave, we have $u(c + x) - u(c) > xu'(c + x)$ and $u(c) - u(c - p) < pu'(c - p)$, which are jointly incompatible with the stated inequality in (8).

4. The Delay-Speedup Asymmetry

A recent study by Loewenstein [1988a] has documented a fourth anomaly, consisting of an asymmetric preference between speeding up and delaying consumption. In general, the amount required to compensate for delaying receiving a (real) reward by a given interval, from t to $t + s$, was from two to four times greater than the amount subjects were willing to sacrifice to speed consumption up by the same interval, i.e., from $t + s$ to t . Because the two pairs of choices are actually different representations of the same underlying pair of options, the results constitute a classic framing effect, which is inconsistent with any normative theory, including DU.

III. A BEHAVIORAL MODEL OF INTERTEMPORAL CHOICE

This section presents a model of intertemporal choice that accounts for the anomalies just enumerated. Our model assumes that intertemporal choice is defined with respect to *deviations* from an anticipated status quo (or "reference") consumption plan; this is in explicit contrast to the DU assumption that people integrate new consumption alternatives with existing plans before making a choice. The objects of choice, then, are sequences of dated adjustments to consumption $\{(x_i, t_i); i = 1, \dots, n\}$, which we shall refer to as *temporal prospects*.

As in the prospect theory for risky choice, we shall represent preference by a doubly separable formula (equation (9) below), which rests on three qualitative properties (see Appendix in Kahneman and Tversky [1979] for details). The first property, also invoked by DU, is that preferences over prospects are intertemporally separable [Debreu, 1959] and can, therefore, be represented by an additive utility function, $\sum_i u(x_i, t_i)$. This important assumption is psychologically most questionable when the choice is perceived to be between complete alternative sequences of outcomes, e.g., savings plans, or multiyear salary contracts. In these cases, it appears that people care about global sequence properties, most notably whether the sequence improves over time [Loewenstein and Prelec, 1991; Loewenstein and Sicherman, 1991]. The present model is primarily concerned with explaining elementary types of intertemporal choices, involving no more than two or three distinct dated outcomes.

In the absence of any strong contrary evidence, we assume that x and t are separable within a single outcome, so that $u(x, t)$

equals $F(v(x)\phi(t))$, where $v(x)$ is a *value function*, $\phi(t)$ a *discount function*, and F an arbitrary monotonically increasing transformation. To eliminate F , one imposes a distributivity condition: (x,t) is indifferent to $(x,t';x,t'')$ implies that (y,t) is indifferent to $(y,t';y,t'')$, for any outcome y , which essentially states that the equality $\phi(t) = \phi(t') + \phi(t'')$ can be established with any one outcome [Kahneman and Tversky, 1979, p. 290]. The discount function is then uniquely specified, given the standard normalization $\phi(0) = 1$. The final model represents preference by the formula,

$$(9) \quad U(x_1, t_1; \dots; x_n, t_n) = \sum_i v(x_i)\phi(t_i).$$

The remainder of this section specifies the properties of the two component functions and shows how the model accounts for the anomalies presented in Section II.

1. Discount Function

The common difference effect reveals that people are more sensitive to a given time delay if it occurs earlier rather than later. Specifically, if a person is indifferent between receiving $x > 0$ immediately, and $y > x$ at some later time s , then he or she will strictly prefer the better outcome if both outcomes are postponed by a common amount t :

$$(10) \quad v(x) = v(y)\phi(s), \quad \text{implies that } v(x)\phi(t) < v(y)\phi(t + s).$$

In order to maintain indifference, the later larger outcome would have to be delayed by some interval s' greater than s . To account for this phenomenon, Ainslie [1975] proposed the discount function, $\phi(t) = 1/t$, which had been found to explain a large body of data on animal time discounting. We now derive a more general functional form, by postulating that the delay that compensates for the larger outcome is a linear function of the time to the smaller, earlier outcome (holding fixed the two outcomes x and y),

$$(11) \quad v(x) = v(y)\phi(s), \quad \text{implies that } v(x)\phi(t) = v(y)\phi(kt + s),$$

for some constant k , which, of course depends on x and y . One can think of this as a more general form of stationarity, in which the "clocks" for the two outcomes being compared run at different speeds. In the normative case, the clocks are identical, and $k = 1$, which yields the exponential discount function [Fishburn and Rubinstein, 1982]. From (11) it follows that

$$(12) \quad v(x)\phi(t') = v(y)\phi(kt' + s),$$

and

$$\begin{aligned}
 (13) \quad v(x)\phi(\lambda t + (1 - \lambda)t') &= v(y)\phi(k(\lambda t + (1 - \lambda)t') + s) \\
 &= v(y)\phi(\lambda(kt + s) + (1 - \lambda)(kt' + s)) \\
 &= v(y)\phi(\lambda\phi^{-1}(v(x)\phi(t)/v(y)) \\
 &\quad + (1 - \lambda)\phi^{-1}(v(x)\phi(t')/v(y))),
 \end{aligned}$$

after substituting for $(kt + s)$ and $(kt' + s)$ from equations (11) and (12). Letting, $r = v(x)/v(y)$, $w = \phi(t)$, $z = \phi(t')$, and $u = \phi^{-1}$, produces a functional equation,

$$(14) \quad ru^{-1}(\lambda u(w) + (1 - \lambda)u(z)) = u^{-1}(\lambda u(rw) + (1 - \lambda)u(rz)),$$

whose only solutions are the logarithmic and power functions: $u(t) = c \ln(t) + d$, $u(t) = ct^\tau + d$ [Aczel, 1966; p. 152, equation (18)]. As $\phi(t) = u^{-1}(t)$, the discount function must be either exponential or hyperbolic.

D1. The discount function is a generalized hyperbola:

$$(15) \quad \phi(t) = (1 + \alpha t)^{-\beta/\alpha}, \quad \alpha, \beta > 0.$$

The α -coefficient determines how much the function departs from constant discounting; the limiting case, as α goes to zero, is the exponential discount function, $\phi(t) = e^{-\beta t}$. Figure I displays the hyperbolic function for three different values of α , along with the pure exponential which is the least convex of the four lines. For each level of α a corresponding β is selected so that the discount function has value 0.3 at $t = 1$. When α is very large, the hyperbola approximates a step function, with value one at $t = 0$, and value 0.3 (in this case) at all other times. This would produce dichotomous time preferences, in which the present outcome has unit weight, and all future events are discounted by a common constant.

As noted already, equation (15) satisfies the empirical "matching law," which integrates a large body of experimental findings pertaining to animal time discounting [Chung and Herrnstein, 1967]; the special case, $(1 + \alpha t)^{-1}$, was proposed initially by Herrnstein [1981], and further investigated by Mazur [1987]; the general hyperbola was defined by Harvey [1986], and given an axiomatic derivation by Prelec [1989] along the lines presented here.

2. Value Function

A distinguishing feature of the current model is the replacement of the utility function with a value function with a reference

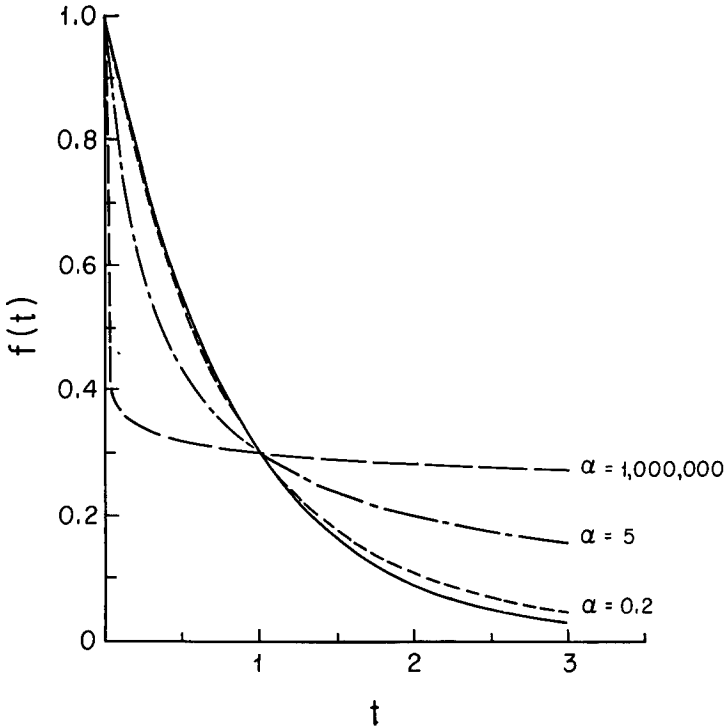


FIGURE I

The Hyperbolic Discount Function $\phi(t) = (1 + \alpha t)^{-\beta/\alpha}$ for Three Different Levels of α . All β s are adjusted so that curves cross at $\phi(1) = 0.3$. The most steeply sloped curve represents conventional exponential discounting.

point, as shown in Figure II. The value function is pieced together from two independent segments, one for losses and one for gains, which connect at the reference point. Such functions have previously been applied to decision making under uncertainty [Kahneman and Tversky, 1979], consumer choice [Thaler, 1980], negotiations [Bazerman, 1984], and financial economics [Shefrin and Statman, 1984]. The shape and reference point assumption reflects basic psychophysical considerations: extra attention to negative aspects of the environment, decreasing sensitivity to increments in stimuli of increasing magnitude, and cognitive limitations.

It is assumed that the reference level represents the status quo (i.e., the current level of consumption), and that new consumption alternatives are evaluated without consideration of existing plans. In certain cases, however, the reference point may deviate from the

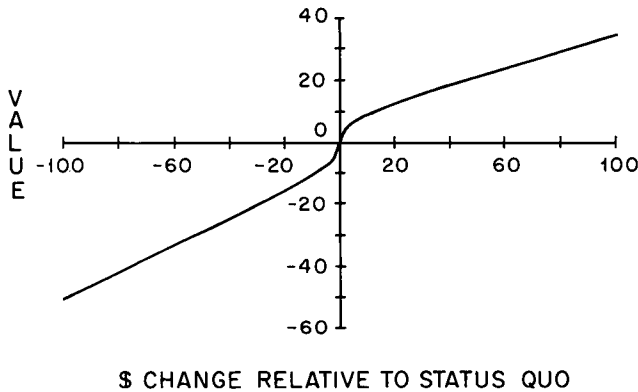


FIGURE II

A Value Function Satisfying the Three Conditions Described in the Text

status quo to reflect psychological considerations such as social comparison [Duesenberry, 1949], or the effect of past consumption which sets a standard for the present [Ferson and Constantinides, 1988; Pollak, 1970].

The function in Figure II is representative of a class of functions that is consistent with the behavioral evidence presented earlier in Section II. The first, and most elementary assumption built into the figure is *loss aversion* [Tversky and Kahneman, 1990].

V1. The value function for losses is steeper than the value function for gains:

$$v(x) < -v(-x).$$

This means that the loss in value associated with a given monetary loss exceeds the gain in value produced by a monetary gain of the same absolute size. In this respect, our value function resembles the prospect theory value function [Kahneman and Tversky, 1979], which also places greater weight on losses.

In the context of intertemporal choice, loss aversion specifically penalizes intertemporal exchanges that are framed in compensating variation terms, i.e., as incurring a loss now in exchange for a future gain, or enjoying a current gain in return for a future loss. For instance, a person who is indifferent between receiving $+q$ now, or $+x$ at some later date, would nevertheless not be willing to

pay q now in order to receive $+x$ at the later date, because the value of $-q$ is greater in absolute magnitude than the value of $+q$.

The remaining two constraints on $v(x)$ are geometrically more subtle and have not been explicitly discussed in the context of prospect theory. Both constraints pertain to the *elasticity* of $v(x)$:

$$(16) \quad \epsilon_v(x) \equiv \frac{\partial \log(v)}{\partial \log(x)} = \frac{xv'(x)}{v(x)}.$$

Our second assumption about the value function is behaviorally determined by the gain-loss asymmetry.

V2. The value function for losses is more elastic than the value function for gains:

$$\epsilon_v(x) < \epsilon_v(-x), \quad \text{for } x > 0.$$

Suppose that $+q$ is the equivalent present value of $+x$ at time t , so that, $v(q) = \phi(t)v(x)$. The gain-loss asymmetry then implies that one would prefer to pay $-q$ now instead of $-x$ at time t : $v(-q) > \phi(t)v(-x)$. Equating $\phi(t)$ in both of these expressions, shows that

$$(17) \quad \frac{v(q)}{v(x)} > \frac{v(-q)}{v(-x)}, \quad \text{for all } 0 < q < x.$$

Consequently, $v(x)$ must “bend over” faster than $v(-x)$, in the precise sense captured by condition V2.³

Our third and final assumption about $v(x)$ is dictated by the magnitude effect, in equivalent variation choices. If $+q$ is the equivalent present value for x at time t , $v(q) = \phi(t)v(x)$, then the magnitude effect predicts that a proportional increase in both q and x , to αq and αx , will cause preference to tip in favor of the later positive outcome, $v(\alpha q) < \phi(t)v(\alpha x)$. As in the previous paragraph, by eliminating $\phi(t)$, we have

$$(18) \quad \frac{v(q)}{v(x)} < \frac{v(\alpha q)}{v(\alpha x)}, \quad \text{for all } 0 < q < x; \alpha > 1.$$

The value function is *subproportional*, like the probability weighting function in prospect theory. As Kahneman and Tversky remarked [1979, p. 282], such a function is convex in log-log

3. Let $u_1(x) \equiv -\ln[v(x)]$ and $u_2(x) \equiv -\ln[-v(-x)]$. Then (17) implies that $u_1(x) - u_1(q) < u_2(x) - u_2(s)$, for all $0 < q < x$, or: $u_1'(x) < u_2'(x)$, for all $x > 0$, which is equivalent to condition V2.

coordinates, which for our model means that the derivative of $\log(v(x))$ with respect to $\log(x)$ is increasing, or that:

V3. The value function is more elastic for outcomes that are larger in absolute magnitude:

$$\epsilon_v(x) < \epsilon_v(y), \quad \text{for } 0 < x < y \text{ or } y < x < 0.$$

The implications of this condition can be visually assessed by comparing Figures II and III. Both figures show the same value function, but plotted over a small (Figure III) or a large (Figure II) range of outcomes. For small outcomes the function is sharply convex, indicating that there is not much perceived value difference between, say, a \$1 gain and a \$2 gain. This property accounts for the high discount rates that apply to small outcomes (i.e., in a choice between \$1 now or \$2 in a year). For large outcomes, however, the function straightens out considerably (Figure II) and, as a result, generates much lower discount rates.

Most probably, the elasticity of the value function does not increase indefinitely, but rather attains a maximum at some large dollar amount, and then begins to decline. When comparing large and unexpected windfalls, it may be reasonable to prefer a million dollars today to several million a few years hence—if drawing on the money in advance was completely prevented. The implausibility of this last requirement makes the interpretation of stated preference over large amounts problematic.

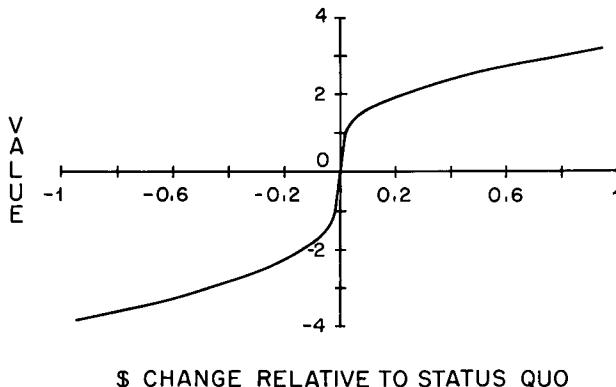


FIGURE III

The Same Value Function as in Figure II, but Plotted over a Smaller Range of Outcomes

IV. FURTHER IMPLICATIONS OF THE MODEL

1. Aversion to Intertemporal Tradeoffs

It follows from our model that a single individual will reveal not one but several discount factors for future cash outcomes, depending on how the choice is formulated. These discount factors can be geometrically derived, as in Figure IV. In the figure we have overlaid the positive and negative branches of the value function, so that both positive and negative outcomes can be represented along the positive x -axis. Starting with a delayed outcome of absolute magnitude x , and a time interval yielding a discount factor for utility of 0.8, we can generate four distinct "present values" for x , depending on whether x is positive or negative, and whether the elicitation method is equivalent or compensating variation. Each present value, divided by x , then yields a specific discount factor.

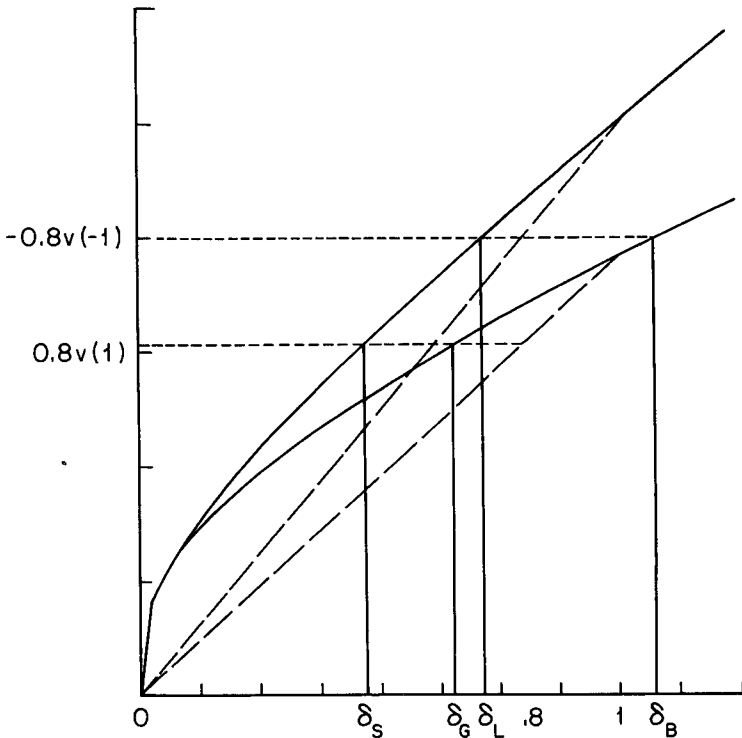


FIGURE IV

Relationship among Discount Factors for Compensating and Equivalent Variation

From equivalent variation, $v(q) = \phi(t)v(x)$, we get the discount factors for gains (G) and losses (L):

$$(19) \quad \delta_{G,L} = \frac{q}{x} = \frac{v^{-1}\{\phi(t)v(x)\}}{x} \quad (\delta_G \text{ for } x > 0, \delta_L \text{ for } x < 0).$$

While from compensating variation, $v(p) + \phi(t)v(x) = 0$, we have the *borrowing* (B) and *saving* (S) factors:

$$(20) \quad \delta_{S,B} = \frac{p}{x} = \frac{v^{-1}\{-\phi(t)v(x)\}}{x} \quad (\delta_S \text{ for } x > 0, \delta_B \text{ for } x < 0).$$

It is apparent from the geometry of the gain and loss value functions in Figure IV, that these discount factors are ordered as $\delta_S < \delta_G < \delta_L < \delta_B$.

A notable aspect of the ranking is the large gap between the savings and borrowing discount factors: a person whose choices are consistent with the value functions in Figure II would require a much more favorable rate in order to borrow than he would to save. The gap between δ_B and δ_S is a measure of how averse a person is to borrowing and savings commitments generally, because it implies a range of risk-free interest rates at which a person will be unwilling to either save or borrow.

The existence of this gap was confirmed by Horowitz [1988], who elicited present and future values for real money payoffs, through a "first-rejected price" auction. According to Horowitz, "The most striking feature of [the] experiment is individuals' apparent aversion to both borrowing and lending." A substantial fraction of subjects revealed discount factors greater than one for borrowing (i.e., they refused zero-interest loans); this, too, is consistent with the model, as we can see from the fact that $\delta_B > 1$ in Figure IV.

2. Framing Effects

As in prospect theory [Kahneman and Tversky, 1979], we assume that the reference level is sensitive to the wording of the questions that elicit the intertemporal tradeoffs. For instance, direct choices between two losses, or two gains, are presumed to be likewise encoded (or "framed") as a pair of positive or negative values. The same would be true of requests for present amounts that create subjective indifference with respect to some future amount of the same sign. In such a context we would interpret the

elicited present value q for amount x at time t , according to the equivalent variation formula, $v(q) = \phi(t)v(x)$.

Questions involving delay or speedup of consumption are a clear case where the compensating variation formula is appropriate. A request, for example, for the maximum value that one would be willing to sacrifice in order to speed up some positive amount (x) from time t to the present, suggests that the baseline levels are zero now, and $+x$ at the future time. In this frame the speedup constitutes a loss of x at time t , and a gain of x minus the speedup cost at time zero. The latter value, p , would then be interpreted according to the compensating variation formula, $v(p) + \phi(t)v(x) = 0$, with $x < 0$ and $p > 0$. The same frame covers delay-of-loss judgments, because in that case there is again a positive present benefit (avoiding the immediate loss), and a future cost (absorbing the loss at the later date). The two complementary question formats—delaying a gain, and speeding up a loss—would yield present values also consistent with equation (20) but for a reversal in the sign of p and x , since there is a negative adjustment to current consumption ($p < 0$), and a positive adjustment to future consumption ($x > 0$).

Figure V compares these predictions with those of the normative model, in which the distinction between a speedup or delay is not recognized. As indicated in the top half of the figure, the discount rates estimated from expediting and delaying gains should be equal, and higher than the devaluation rates estimated from expediting and delaying losses. In contrast, the reference point model predicts that common rates will be observed for the diagonal pairs in the matrix, with the delaying gains/speeding up losses pair producing a higher estimate.

Clear support for the reference point model can be found in the data reported by Benzion et al. [1989]. Figure VI displays implicit discount rates calculated from their data for each of the four elicitation methods. As predicted, discount rates are high and virtually identical for expediting a loss (white diamonds) and delaying a gain (black squares), and lower and again virtually identical for expediting a gain (black triangles) and delaying a loss (white squares).

Our second framing example is produced by the discrepancy between discounting of gains and losses. In this study 85 students in an MBA class on decision making were randomly divided into two groups which each answered one of the following two questions.

	Speed-up	Delay
Gains	High	High
Losses	Low	Low

Normative Model

	Speed-up	Delay
Gains	High	Low
Losses	Low	High

Reference Point Model

FIGURE V

Discount Rates When Expediting and Speeding up Gains and Losses: Comparison of DU and Reference Point Model Predictions

Version 1.

Suppose that you bought a TV on a special installment plan. The plan calls for two payments; one this week and one in six months. You have two options for paying: (circle the one that you would choose)

- A. An initial payment of \$160 and a later payment of \$110.
- B. An initial payment of \$115 and a later payment of \$160.

Version 2.

Suppose that you bought a TV on a special installment plan. The plan calls for two payments of \$200; one this week and one in six months. Happily, however, the company has announced a sale which applies retroactively to your purchase. You have two options: (circle the one that you would choose)

- C. A rebate of \$40 on the initial payment and a rebate of \$90 on the later payment.
- D. A rebate of \$85 on the initial payment and a rebate of \$40 on the later payment.

Since options A and C and options B and D are the same in

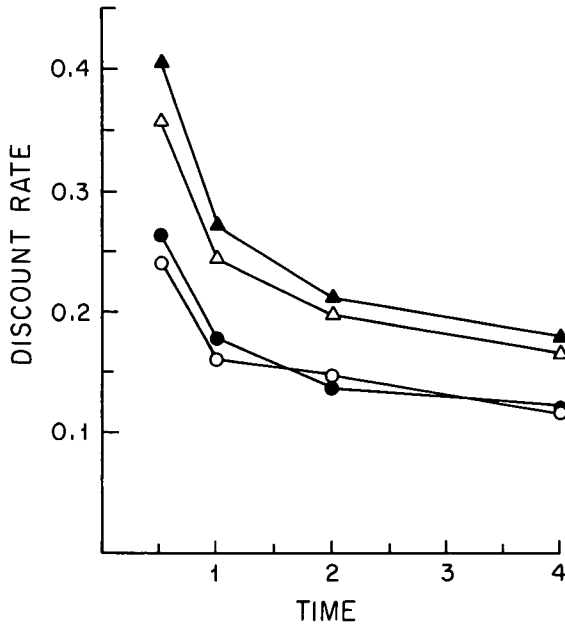


FIGURE VI

Implicit Discount Rates from Benzion et al. [1989]

The rates have been averaged across the four dollar amounts used in their study.

terms of payoffs and delivery times, DU predicts that there will be no systematic difference in responses to the two versions. Nevertheless, a higher fraction of subjects opted for the lower-discount option (the one involving greater earlier payments) when the question was framed as a loss rather than as a gain. Fifty-four percent of subjects exposed to version 1 stated a preference for A over B. However, a significantly different fraction (33 percent) preferred C over D ($X^2(1) = 3.9, p < 0.05$). The proposed model explains the observed pattern of responses as follows: in the first frame the large, negative outcomes suffer less discounting, which causes people to decide on the basis of total payments. In the second frame, however, the outcomes are smaller in absolute magnitude and positive. Both of these factors contribute to relatively high discounting of the delayed outcomes, leading to a preference for the second option which offers a greater initial rebate.

The choice of appropriate frame is not always unambiguous. A savings decision, for example, can be viewed as a simple choice

between benefits enjoyed now or later (equation (19)) or a postponement of present consumption for the future (equation (20)). Such changes in frame will, according to our theory, affect the range of interest rates that a person considers acceptable.

3. Effect of Prior Expectations on Choice

Consider two people waiting for an object (e.g., a computer): one has been told to expect delivery in two weeks; the other anticipates delivery in four weeks. Two weeks pass, and both are faced with a new choice: the original computer to be delivered immediately, or a superior computer to be delivered in two weeks. Who is more likely to wait? If both parties adapt their reference points to anticipated delivery times, then the reference point model predicts that the person who anticipated delivery in two weeks will be more impatient. This person frames the choice as the status quo versus a loss of a computer immediately and a gain of a slightly superior computer in two weeks. Loss aversion and discounting both mitigate against choice of the delayed, superior, model. On the other hand, the person who anticipated later delivery would frame the choice as a loss of the later computer and gain of an earlier computer. Here loss aversion discourages the choice of the earlier computer, while discounting has an opposing influence. Thus, we predict that the person anticipating two-week delivery would be more likely to accept. In effect, people who are psychologically prepared for delay are more willing to wait.

This prediction was tested in a laboratory experiment conducted with 105 suburban Chicago tenth graders [Loewenstein, 1988b]. All prizes were in the form of nontransferable gift certificates. As a result of an earlier experiment, half the students expected to obtain a \$7 gift certificate at an earlier date, half at a later date. When the earlier date arrived, all subjects were given a new choice between getting the \$7 certificate immediately or a larger valued certificate at a later date. As predicted, prior expectations had a significant impact on choice. Twenty-seven out of 47 subjects who anticipated getting the prize at the earlier date opted for the immediate \$7; only 17 of the 57 who expected late delivery chose not to wait for the larger prize, a statistically significant difference.

4. High Discount Rates Estimated from Purchases of Consumer Durables

Several studies have estimated discount rates from purchases of consumer durables (e.g., air conditioners) [Hausman, 1979;

Gately, 1980]. Such purchases typically involve an up-front charge (the purchase price) and a series of delayed charges (e.g., electricity charges). Because more expensive models are generally more energy efficient, it is possible to calculate the discount rate (or range of discount rates) implicit in a particular purchase. A second source of behavioral estimates of discount rates has been the studies of major economic decisions such as saving [Landsberger, 1971] and intertemporal labor-leisure substitution [Hotz, Kydland, and Sedlacek, 1988; Moore and Viscusi, 1988].

The estimates from these two classes of studies have differed sharply. Studies of consumer durable purchases show very high average discount rates (across different income groups), e.g., from 25 percent [Hausman, 1979] to 45–300 percent [Gately, 1980]. Research on savings behavior or labor supply has almost uniformly found much lower discount rates (typically well below 25 percent). How can these estimates be reconciled? The proposed model predicts that the small delayed electricity charges associated with the consumer durables will be substantially devalued due to the dependence of discounting on outcome magnitude. Thus, consumer durable purchases will be insensitive to electricity charges, and discount rates estimated from those purchases will appear to be high. Discount rates estimated from major economic decisions would not be subject to such small-magnitude effects.

5. *Nonmonotonic Optimal Benefit Plans*

Our model makes certain predictions about the shape of the optimal intertemporal allocation of benefits under a constant market present value constraint. Assuming that consumption at a point in time, $x(t)$, is framed as a positive quantity, the value of the plan, covering the period from 0 to T , is given by the continuous version of the discounted value formula,

$$(21) \quad \int_0^T \phi(t)v(x(t)) dt.$$

The optimal plan $x^0(t)$, given a market interest rate r and a present value constraint,

$$(22) \quad \int_0^T e^{-rt}x(t) dt \leq I,$$

can be calculated by standard techniques [Yaari, 1964]. Yaari showed that if the optimal plan exists, and if the value function is concave and continuously differentiable, then the rate of change in

consumption, for the optimal plan, equals [equation (21)]:

$$(23) \quad \frac{\partial x^0}{\partial t} = r - \left(-\frac{\phi'(t)}{\phi(t)} \right) \left(-\frac{v'(x^0(t))}{v''(x^0)} \right).$$

As Yaari observed, the direction of local change in consumption rate is controlled by the sign of the difference between the market interest rate and the rate of time preference ($-\phi'/\phi$). In view of our hyperbolic discounting assumption, this allows for only three qualitatively distinct possibilities: (1) the rate of time preference is always greater than the market rate, in which case consumption is decreasing throughout the interval; (2) the rate of time preference is always lower than the market rate, in which case consumption increases over the interval; (3) the rate of time preference starts off above the market rate, but eventually drops below it and remains so, in which case consumption will decline to a minimum value (when the two rates equalize) and then increase afterwards.

Relative to normative theory, our model suggests that people may tend to prefer plans that sacrifice the medium-range future for the sake of the short and the long term. There is nothing clearly wrong with this, provided that one can commit to an entire plan at the moment of decision. However, if the optimal plan can be recalculated at later points in time, then the planned sacrifice in midrange consumption will not take effect [Strotz, 1956]. As a result, a bias in favor of the long and short runs may in practice yield behavior that is oriented only to the short run.

This discussion presupposes a concave value function, which—although not explicitly assumed in V1–V3—is certainly true for the function in Figure IV. In the loss domain, however, our working assumption is that the value function is convex, at least initially, which means that the most attractive plan for intertemporal loss allocation consists of concentrating the loss at a single point in time. The (negative) value of the loss, if allowed to accumulate at the market rate to time t , equals $\phi(t)v(Ie^{rt})$, which means that it will pay to delay payment whenever $\phi'(t)v(Ie^{rt}) + rIe^{rt}\phi(t)v'(Ie^{rt}) > 0$ or, after rearranging, whenever

$$(24) \quad r < \frac{-\phi'(t)/\phi(t)}{\epsilon_v(Ie^{rt})}.$$

The product on the right is decreasing in t , since $-\phi'/\phi$ equals $\beta/(1 + \alpha t)$ by Assumption D1, and $\epsilon_v(Ie^{rt})$ is increasing by Assumption V3. Hence there is a unique point in time—possibly at one or

the other endpoint of the interval—at which the loss is absorbed with smallest perceived cost.

6. *Other Predictions*

Our model has several implications for the behavior of key economic variables during business cycles. First, it predicts that psychological factors will amplify the tendency for businesses to cut back on investment during periods of lower than anticipated profits. In high profit periods the investment project is viewed in terms of equivalent variation, as a choice between two gains: take the excess profit now, or take greater profits from investment later. But in periods of low or negative returns, an identical investment opportunity would be viewed in terms of compensating variation, i.e., as incurring a current loss in exchange for a future gain, which, as shown in the previous section, will induce a higher subjective discount rate. There may, of course, be good economic reasons for reducing investments during economic downturns; what the model suggests is that psychological factors additionally and independently contribute to the reduction.

For consumers too, an economic downturn should cause an increase in impatience and a consequent decrease in saving. Consumers are likely to frame drops in disposable income, or negative departures from expected gains, as losses, so that saving from income will be viewed in terms of compensating variation: a further loss in the present for a gain in the future.⁴ Saving out of an expanding income or out of bonus income is more likely to be viewed in terms of compensating variation, inducing lower discounting and greater saving. Consistent with this prediction, there is evidence that the marginal propensity to save income from bonuses is higher than that from normal income [Ishikawa and Ueda, 1984].

Our model is also possibly relevant to the so-called “disposition effect” in real estate [Case and Shiller, 1989] and financial markets [Shefrin and Statman, 1985; Ferris, Haugen and Makhija, 1988]. This effect refers to the fact that people tend to hold on to losing stocks and to real estate that has dropped in value, which depresses trading volume during market downturns. In such

4. The low rates of savings and negative real rates of interest in the 1970s [Mishkin, 1981] may reflect the shortfall from expectations induced by economic stagnation following the prolonged economic boom of the 1960s. At a societal level the tax cuts of the early 1980s, which entailed a transfer of income from the future to the present, can be interpreted similarly.

situations people have a choice between taking an immediate loss (by selling) or holding on to the asset with the potential of further loss or potential gain. Since the value function is convex in the loss domain, further losses are less than proportionately painful, while gains yield marginally increasing returns. The incentives are thus stacked in favor of holding on to the asset. The incentives are reversed on the gain side, motivating people to quickly sell assets that have gained in value.

In general, the market level implications of the model depend critically on the presence or absence of arbitrage opportunities that exist in a particular economic domain. Arbitrage opportunities are extensive in some markets, such as those for fixed rate financial assets where leveraged short sales are possible. In other markets, e.g., labor markets, arbitrage opportunities are virtually nonexistent. We would expect to see the effects of subjective time discounting manifested more clearly in the latter markets, in the specific case through labor contracts that offer large initial wage increases.

In financial markets the effects of scale and sign, produced by the curvature of the value function, will presumably be arbitrated away. If a particular market were to offer high interest rates on small investments, reflecting the magnitude effect, investors would simply borrow large sums and then invest them in small packages, driving down the rate on small investments.

Hyperbolic discounting is less easily arbitrated, even in financial markets. If most people demanded lower rates of return for long investment periods than for short ones, the yield curve would be downward sloping with no opportunities for arbitrage. Those who discounted the future at a constant rate would tend to invest in short-term securities, and might even short the long-term securities, but they could not do so without risk. Without denying that many purely economic factors influence the yield curve, our model suggests that psychological biases will independently exert pressure toward downward sloping.⁵

V. CONCLUDING REMARKS

The discounted utility model has played a dominant role in economic analyses of intertemporal choice. Although economists have experimented with alternative formulations, these efforts

5. Our analysis may help to explain Fama's [1984] finding that, contrary to the liquidity preference hypothesis, the yield curve tends to drop, on average, past a certain point.

have typically responded to a single limitation of DU (e.g., increasing consumption postretirement) rather than to a more comprehensive critique. DU's basic assumptions and implications have, for the most part, not been questioned. This paper presents an integrated critique of DU, enumerating a series of intertemporal choice anomalies that run counter to the predictions of DU.

Perhaps most importantly, sensitivity to time delay is not well expressed by compound discounting. A given absolute delay looms larger if it occurs earlier rather than later; people are relatively insensitive to changes in timing for consumption objects that are already substantially delayed. Second, the marginal utility of consumption at different points in time depends not on absolute levels of consumption, but on consumption relative to some standard or point of reference. Generally, the status quo serves as reference point; people conserve on cognitive effort by evaluating new consumption alternatives in isolation, rather than by integrating them with existing plans.

Our model by no means incorporates all important psychological factors that influence intertemporal choice. For example, like any model with nonconstant discounting, it yields time-inconsistent behavior or "myopia" as Strotz [1955] called it. However, it cannot explain the high levels of conflict that such myopic behavior often evokes. Intertemporal choice often seems to involve an internal struggle for self-command [Schelling, 1984]. At the very moment of succumbing to the impulse to consume, individuals often recognize at a cognitive level that they are making a decision that is contrary to their long-term self-interest. Mathematical models of choice do not shed much light on such patterns of cognition and behavior (but see Ainslie [1985]).

Such episodes of internal conflict are not entirely random. Certain types of situations, such as when a person comes into direct sensory contact with a choice object, seem to elicit especially high rates of time discounting, while others do not. People exhibit high rates of discounting when driven by appetites such as hunger, thirst, or sexual desire. While not incompatible with the present model, these phenomena are not predicted by it.

Finally, our model does not incorporate preference interactions between periods, despite the fact that our own recent empirical research has shown such interactions to be pervasive when people choose between sequences of outcomes. Preference interactions are revealed through a strong dislike of deteriorating outcome sequences, and through a liking for evenly spreading

consumption over time [Loewenstein and Prelec, 1991]. A taste for steady improvement seems to capture the preferences of most subjects, when sequences are being considered. Generally, the present model is more applicable to short-range decisions involving simple outcomes rather than long-term planning of consumption. No simple theory, however, can hope to reflect all motives that influence a particular decision. We have attempted to demonstrate that a theory with only two scaling functions can explain much of the observed deviation in preference from the normative discounted utility model.

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